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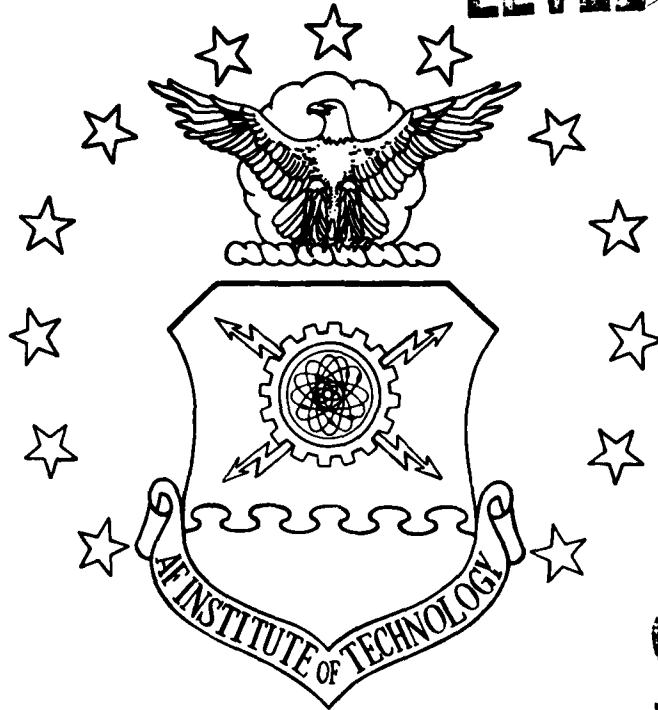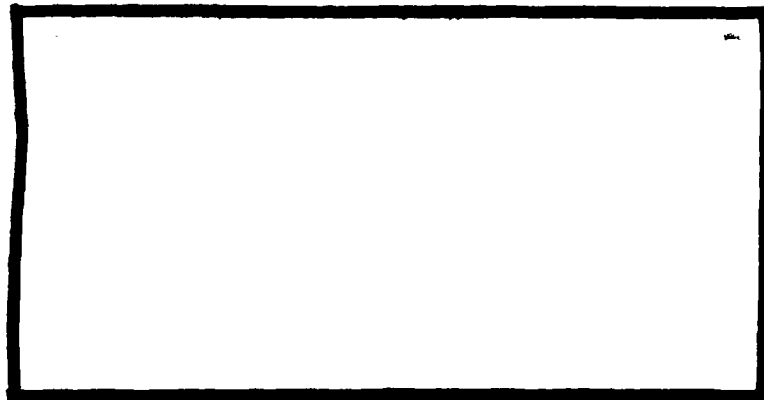
DEPARTMENT OF THE AIR FORCE

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AIR FORCE INSTITUTE OF TECHNOLOGY

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A PERTURBATION ANALYSIS OF MODAL

CONTROL OF AN UNSTABLE PERIODIC ORBIT

THESIS

AFIT/GA/AA/81D-4   Dennis W. Ehrler
                         Captain   USAF

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THESIS

Presented to the Faculty of the School of Engineering

of the Air Force Institute of Technology

Air University

in Partial Fulfillment of the

Requirements for the Degree of

Master of Science

by

Dennis W. Ehrler

Captain   USAF

Graduate Astronautical Engineering

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Contents

iii

List of Figures

List of Tables

List of Symbols

A	coefficient matrix of the variational system
$\overline{B}$	control vector
CT.	control term
$\overline{E}$	eigenvectors of the four-body system
e	exponentiation
$\overline{f}$	partial deri tives of the Hamiltonian
G	universal gravitational constant
H	Hamiltonian of the four-body system
H.O.T.	higher order terms
I	identity matrix
J	Jordan form of the matrix of Poincare exponents
k	controller gain
L1-L5	Lagrange points of equilibrium in earth-moon system
Λ	matrix of eigenvectors
λ	Poincare exponents of the four-body system
M	sum of the masses of the earth, moon, and sun
m_1	mass of the earth
m_2	mass of the moon
m_3	mass of the sun
m	mass of the satellite
$\overline{n}$	modal vector of the four-body problem

$\bar{n}_p$	particular solution to the modal vector differential equation
p_x, p_y, p_z	momenta of the satellite
$\bar{P}$	first order perturbation caused by the moon's orbital eccentricity
Φ	state transition matrix
q_x, q_y, q_z	coordinates of the satellite position
$\bar{R}$	vector from the earth-moon center of mass to the satellite
$r\dot{\bar{R}}$	time derivative of $\bar{R}$ taken with respect to the rotating reference frame
$\bar{r}$	vector from the earth to the moon
$\bar{r}_{sat}$	inertial position vector of the satellite
$\bar{\rho}$	vector from the earth-moon center of mass to the sun
$r\dot{\bar{\rho}}$	time derivative of $\bar{\rho}$ taken with respect to the rotating frame
t	time
t_0	initial time
T	kinetic energy of the satellite
T_s	specific kinetic energy of the satellite
τ	period of the orbit about L3
$\bar{u}$	control vector
V	potential energy of the satellite
V_s	specific potential energy of the satellite
$\bar{V}_s$	velocity vector of the satellite

$\bar{\omega}$	angular velocity vector of the rotating reference frame
$\bar{x}$	state vector of the satellite
$\bar{x}_h$	homogeneous solution to the state vector differential equation
$\bar{x}_p$	particular solution to the state vector differential equation
$\bar{x}_n$	solution state vector
$\delta \bar{x}$	variational state vector

Abstract

A Fourier representation of the perturbation effects of the moon's eccentricity on a satellite in orbit about L3 was constructed and verified numerically. It was then incorporated in a modal control scheme for linear systems that are time periodic which was developed by Shelton (Ref. 5). The feasibility of this modified control scheme was then verified by computer simulation by controlling the actual non-linear motions of the satellite with the modified controller.

A PERTURBATION ANALYSIS OF MODAL

CONTROL OF AN UNSTABLE PERIODIC ORBIT

I. Introduction

Background

Space is fast becoming a part of our everyday lives.
Sensor-bearing satellites placed in orbit around the earth
play a very important role in the world we live.   From ter-
rain and resource mapping to communication and strategic
reconnaisance our lives are all affected by our exploits in
space.

A desire to use the minimum number of satellites to
satisfactorily accomplish mission objectives and maintain the
orbits of these satellites has led to much research attemp-
ting to develop better ways to accomplish this task.   A num-
ber of space operations require total coverage of the earth's
surface by a particular set of satellites.   Currently, such
space operations are conducted by satellites positioned in
earth-synchronous orbits, but not only does this not provide
total global coverage, these orbits are unstable and require
some sort of orbit maintaining device to be carried by the
satellites. This control system, and the fuel to power it,
must be carried as an integral part of the satellite, taking
away, pound for pound, available payload weight allocated to

the sensors and support equipment required to perform the satellite's primary mission. Also, when the control system's fuel supply is exhausted, the satellites service life is at an end. These factors combine to force design tradeoffs between service life versus mission capability.

In recent years global strategic reconnaisance and communication have become increasingly important and, at the same time, more vulnerable to disruption. With the Soviet Union's increased research and development efforts aimed at developing "satellite killers" a less vulnerable position for these satellites is desirable.

The search for new orbits which would provide a maximum amount of, or total, earth coverage, hold down the control cost of maintaining these orbits and still decrease vulnerability to attack has led to extensive research into the "Lagrange" points. The five Lagrange points of equilibrium shown in Figure 1 represent theoretical points in space, in a coordinate system that rotates with the earth-moon system, where the combined gravitational effects of the earth and moon will keep a satellite placed at one of these points at that point. However, only L4 and L5 are stable and would provide long term stable orbits. Since the Lagrange points lie in the earth-moon plane and not in the earth's equitorial plane, one satellite placed at each of the points L3, L4, and L5 would provide "global" coverage, including the strategically valuable North Pole, at all times.
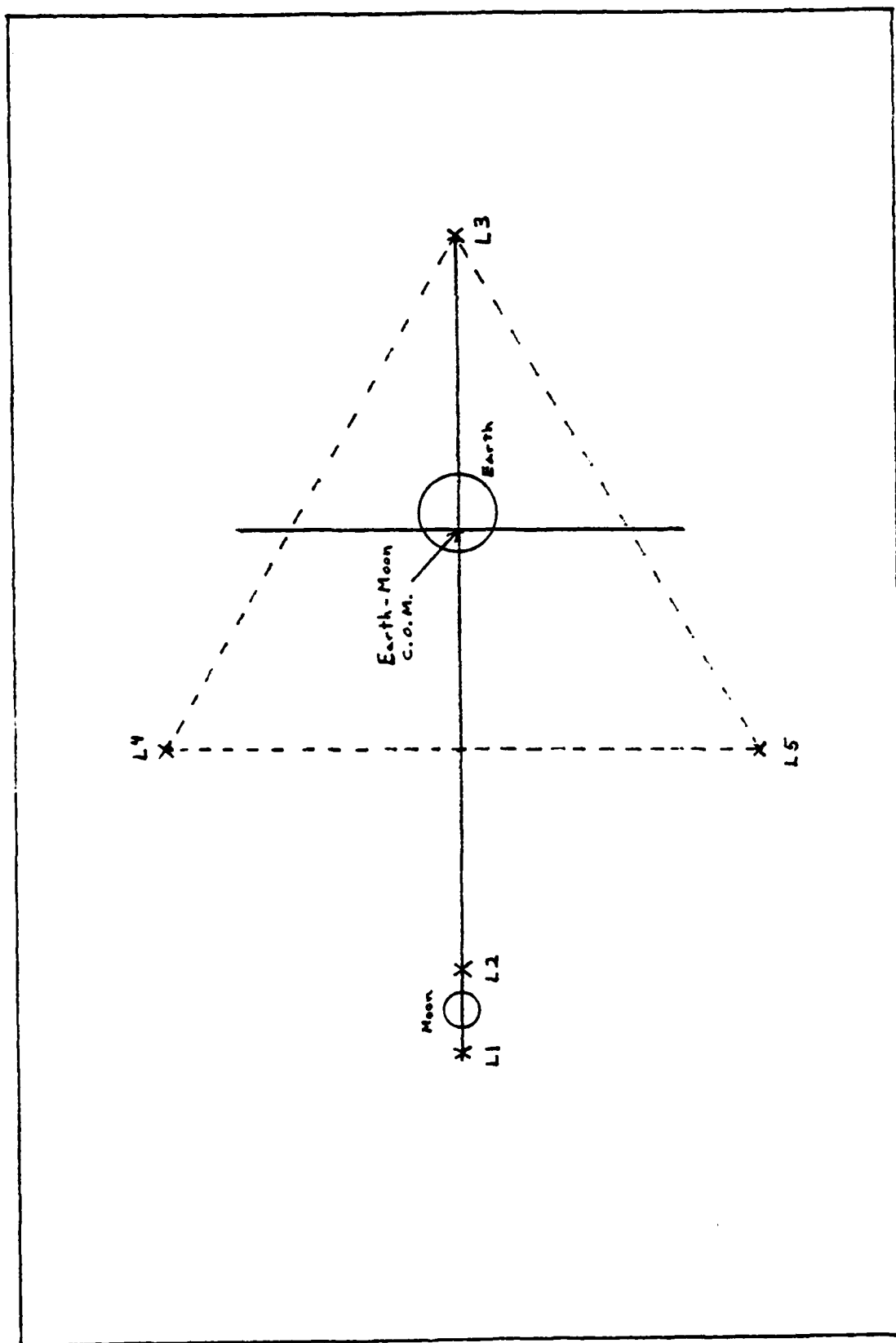
Figure 1. Lagrange Points in Earth-Moon System

3

Since the analysis from which the Lagrangian points are derived is based on the assumptions of circular, planar orbits and does not include the sun, when the "restricted four body problem" is analyzed all of the Lagrangian points become unstable. However, the points do provide a place to start.

Problem and Scope

Shelton developed a modal control scheme for linear systems that are time periodic. This scheme was then applied to a satellite in orbit about the earth-moon Lagrange point, L3. However, his results are only valid for a set of dynamics based upon his specific periodic orbit. The free lunar eccentricity, the lunar inclination, and the eccentricity of the sun were not included in his analysis.

Since the free lunar eccentricity is the major contributor of in-plane perturbations about the periodic orbit, this study will focus on its effects. With the active controller developed by Shelton operating, the periodic orbit becomes a stable reference solution and classical perturbation techniques can be applied to the addition of the free lunar eccentricity. If the solution, including the first order perturbation terms, is well behaved, free of resonances and unstable roots, then the controller will be modified to suppress only the "free" components causing the original instability. Higher order terms will hopefully be several orders of magnitude smaller than the first order terms

leaving us to believe if the controller operates satisfactorily in the lowest order perturbed system that the higher order terms are negligible. Applying this to Shelton's system we will again arrive at a control system with effectively zero long-term stationkeeping costs, but that is valid in the "real" representation of the dynamics and not just in the idealized system.

II. Problem Analysis

Modeling Assumptions

Shelton's research was based on a model of the three attracting bodies (earth, moon, and sun) that placed the moon in an orbit about the earth that is periodic but not circular. The earth orbit around the sun was modeled as circular and other inclination and eccentricity effects were initially ignored, restricting the motions of the four bodies to a plane. This study used these assumptions to rederive the Shelton controller followed by a perturbation analysis.

Coordinate System

Due to the nature of the periodicity of the orbit about L3, the coordinate system used in Shelton's research, and in this study, rotates with the average angular velocity of the moon and the center of the system is located at the earth-moon center of mass. This frame allows easy visualization of the periodic nature of the orbit about L3 (Ref. 6).

Derivation of the Hamiltonian (Ref. 6)

In deriving the dynamics for the restricted four-body problem, the three attracting bodies were assumed to have their actual masses while the satellite was assumed to be massless. This assures that the attracting bodies affect the motion of the satellite, but the satellite has no effect on
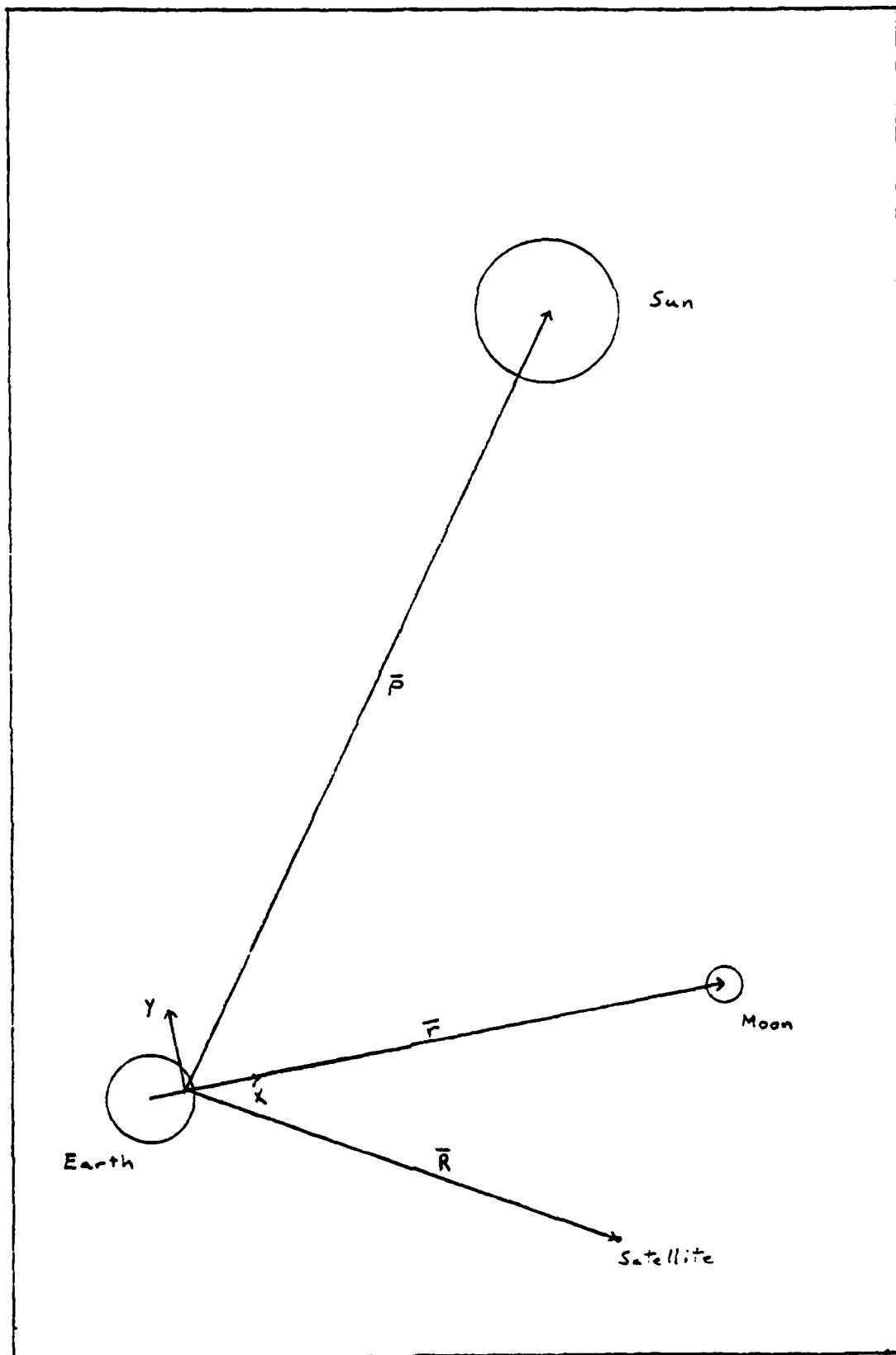
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Figure 2. Geometry of the Four-Body Problem

7

the motion of the attracting bodies. The geometry of this system is depicted in Figure 2.

In order to derive the Hamiltonian the kinetic and potential energies must first be constructed. The inertial point in this derivation is assumed to be the earth-moon-sun center of mass. This results in the inertial position vector of the satellite to be

$$\overline{r}_{Sat} = \overline{R} - \frac{m_3}{M} \overline{\rho} \tag{1}$$

where $M = m_1 + m_2 + m_3$ (sum of the masses of the earth, moon, and sun, respectively) and the factor m_3/M which locates the inertial point of the system. Taking the inertial time derivative of the position vector in the rotating frame yields

$$\frac{^i d}{dt}(\overline{r}_{Sat}) = \overline{v}_{Sat} = \overset{r.}{\overline{R}} + (\overline{\omega} \times \overline{R}) + (-\frac{m_3}{M}) \overset{r.}{\overline{\rho}} + \overline{\omega} \times (-\frac{m_3}{M}) \overline{\rho} \tag{2}$$

where $-\frac{m_3}{M} \overset{r.}{\overline{\rho}} + \overline{\omega} \times (-\frac{m_3}{M}) \overline{\rho}$ is the transport velocity of the earth-moon center of mass. The symbol $\overline{\omega}$ denotes the angular velocity vector of the rotating frame, which is the moon's mean inertial angular velocity vector, and the superscript r represents differentiation in the rotating frame. The kinetic energy, T, of the satellite is defined as

$$T = 1/2 \ m \ (\overline{V}_{Sat} \cdot \overline{V}_{Sat}) \tag{3}$$

Because the satellite is assumed to be massless, the mass is divided from both sides which gives us the specific kinetic energy, T_s. Solving for T_s from equations (2) and (3) yields

$$T_s = 1/2 \{ \dot{\bar{R}} \cdot \dot{\bar{R}} + (\bar{\omega} \times \bar{R}) \cdot (\bar{\omega} \times \bar{R}) + \frac{-m_3}{M} \dot{\bar{\rho}} \cdot \frac{-m_3}{M} \dot{\bar{\rho}}$$

$$+ (\bar{\omega} \times \frac{-m_3}{M} \bar{\rho}) \cdot (\bar{\omega} \times \frac{-m_3}{M} \bar{\rho}) + 2[\dot{\bar{R}} \cdot (\bar{\omega} \times \bar{R})] + 2(\dot{\bar{R}} \cdot \frac{-m_3}{M} \dot{\bar{\rho}})$$

$$+ 2[\dot{\bar{R}} \cdot (\bar{\omega} \times \frac{-m_3}{M} \bar{\rho})] + 2[(\bar{\omega} \times \bar{R}) \cdot \frac{-m_3}{M} \dot{\bar{\rho}}] + 2[(\bar{\omega} \times \bar{R}) \cdot (\bar{\omega} \times \frac{-m_3}{M} \bar{\rho})]$$

$$+ 2[\frac{-m_3}{M} \dot{\bar{\rho}} \cdot (\bar{\omega} \times \frac{-m_3}{M} \bar{\rho})] \} \tag{4}$$

The potential energy, V, of the satellite due to the gravitational attraction of the earth, moon, and sun is

$$V = \sum_{i=1}^{3} \frac{-Gm_i m}{r_i} \tag{5}$$

where m is the mass of the satellite, r_i is the magnitude of the vector from the ith body to the satellite, and G is the universal gravitational constant. Dividing both sides of (5) by the mass of the satellite gives us the specific potential energy, V_s.

$$V_s = \frac{-Gm_3}{|\bar{R} - \bar{\rho}|} - \frac{Gm_2}{|\bar{R} - \frac{m_1}{m_1+m_2}\bar{r}|} - \frac{Gm_1}{|\bar{R} + \frac{m_2}{m_1+m_2}\bar{r}|} \tag{6}$$

as the potential energy is not dependent on the time deriva-
tives of the coordinates, the momenta, p_i, of the Hamiltonian
are

$$p_i = \frac{\partial T_s}{\partial \dot{q}_i} \tag{7}$$

where $\dot{q}_i$ is the time derivative of the ith coordinate. The
coordinates used were the x, y, and z components of the
satellite's position as depicted in Figure 2. In this frame
the momenta, p_i, are the actual inertial velocity components,
a fact that Shelton used to derive his controller.

The Hamiltonian is given by

$$H = \sum_{i=1}^{3} p_i \dot{q}_i - T_s + V_s \tag{8}$$

Combining equations (4), (6) and (7) gives us the Hamiltonian.

$$H = \frac{1}{2}(p_x{}^2 + p_y{}^2 + p_z{}^2) + \omega (p_x R_y - p_y R_x) + \frac{m_3}{M}[(\dot{\rho}_x - \omega \rho_y)p_x$$

$$+ (\dot{\rho}_y + \omega \rho_x)p_y] - \frac{Gm_3}{|\bar{R} - \bar{\rho}|} - \frac{Gm_2}{|\bar{R} - \frac{m_1}{m_1+m_2}\bar{r}|} - \frac{Gm_1}{|\bar{R} + \frac{m_2}{m_1+m_2}\bar{r}|} \tag{9}$$

From this Hamiltonian the equations of motion of the

satellite were formed.

Equations of Motion

The equations of motion for the satellite are derived from the Hamiltonian by forming

$$\dot{q}_m = \frac{\partial H}{\partial p_m} \qquad m=1,2,\ldots,n \tag{10}$$

$$\dot{p}_m = \frac{\partial H}{\partial q_m} \qquad m=1,2,\ldots,n \tag{11}$$

where n represents the number of generalized coordinates describing the system (Ref. 5:94). These equations become

$$\dot{\overline{x}}(t) = \overline{f}(\overline{x}(t),t) \tag{12}$$

when put in state variable form. The state vector $\overline{x}$ is made up of the coordinates and momenta, q's and p's, and $\overline{f}$ contains the partial derivatives of the Hamiltonian defined by (10) and (11). In this restricted four-body problem the Hamiltonian is an explicit function of time so the equations of motion are explicit functions of time also.

Shelton assumed the existence of a periodic solution to (12), $\overline{x}_0(t)$. Defining

$$\delta\overline{x}(t) = \overline{x}(t) - \overline{x}_0(t) \tag{13}$$

11

and differentiating (13) to get

$$\delta \dot{\overline{x}}(t) = \dot{\overline{x}}(t) - \dot{\overline{x}}_0(t) \tag{14}$$

Substituting (13) and (14) into (12) results in

$$\dot{\overline{x}}_0(t) + \delta \dot{\overline{x}}(t) = \overline{f}(\overline{x}_0(t) + \delta \overline{x}(t), t) \tag{15}$$

Expanding this equation in a Taylor Series about $\overline{x}_0(t)$ produces

$$\dot{\overline{x}}_0(t) + \delta \dot{\overline{x}}(t) = \overline{f}(\overline{x}_0(t), t) + \frac{\partial \overline{f}}{\partial \overline{x}(t)} \bigg|_{\overline{x}_0(t)} \delta \overline{x}(t) + H.O.T. \tag{16}$$

but from (12) we see that

$$\dot{\overline{x}}_0(t) = \overline{f}(\overline{x}_0(t), t) \tag{17}$$

which reduces (16) to

$$\delta \dot{\overline{x}}(t) = A(t) \delta \overline{x}(t) \tag{18}$$

including only first order terms, with

$$A(t) = \frac{\partial \overline{f}}{\partial \overline{x}(t)} \bigg|_{\overline{x}_0(t)} \tag{19}$$

12

Displacing the periodic solution, $\bar{x}_0(t)$, at t_0 an amount, $\delta\bar{x}$, and expanding the solution in a Taylor series about $\bar{x}_0(t)$ yields

$$\bar{x}(t) = \bar{x}_0(t) + \frac{\partial\bar{x}(t)}{\partial\bar{x}(t_0)}\bigg|_{\bar{x}_0(t)} \delta x(t_0) + H.O.T. \tag{20}$$

Truncating to first order and using (13) produces

$$\delta\bar{x}(t) = \Phi(t,t_0)\ \delta\bar{x}(t_0) \tag{21}$$

where

$$\Phi(t,t_0) = \frac{\partial\bar{x}(t)}{\partial\bar{x}(t_0)}\bigg|_{\bar{x}_0(t)} \tag{22}$$

The matrix, Φ, is called the state transition matrix and is a function of only the initial and final times being considered. The properties of the Φ matrix are

$$\Phi(t,t_0) = \Phi(t,t_1)\ \Phi(t_1,t_0) \tag{23}$$

and

$$\Phi(t_0,t_0) = I \tag{24}$$

where I is the Identity matrix. Differentiating (21) with respect to time produces

13

$$\delta\dot{\overline{x}}(t) = \dot{\Phi}(t,t_0) \; \delta\overline{x}(t_0) \qquad (25)$$

as $\delta\overline{x}(t_0)$ is a constant.

Floquet Theory

The four-body problem Hamiltonian, derived in a previous section, is an explicit function of time and not a constant of the motion. Energy is not conserved and no other constants of the motion exist (Ref. 5:416). The equations of motion that are derived from the Hamiltonian are also time varying. The variational equations of motion are written in state vector form, derived previously, as

$$\delta\dot{\overline{x}}(t) = A(t) \; \delta\overline{x}(t) \qquad (26)$$

This system is periodic and the periodicity is contained in the coefficient matrix $A(\tau)$ where τ is defined as the period of the system. Differential equations such as (26), that have periodic coefficients, can be reduced to the case of constant coefficient, linear differential equations, by the theory of Floquet (Ref. 3:60). Floquet's theory states the general solution to (26) is of the form

$$\delta\overline{x}(\tau) = \sum_{j=1}^{n} \beta_j \overline{E}_j(t) e^{\lambda_j t} \qquad (27)$$

where β_j and λ_j are constants and the $\overline{E}_j$ are periodic func-

tions with the same period, τ. The n λ_j can be arranged in a matrix of the Jordan canonical form which basically diagonalizes the matrix (Ref. 5:267). The constant λ_j is called a characteristic or Poincare exponent of A(t).

In order to compute $\overline{E}_j$ and λ_j Shelton assumed that he needed to excite one mode of the system described by (26). Letting $\beta_j = 0$ for $j \neq i$ and $\beta_i = 1$ yields, from (27),

$$\delta \overline{x}(t) = \overline{E}_i(t) e^{\lambda_i t} \tag{28}$$

From (21), with $t_0 = 0$

$$\delta \overline{x}(t) = \Phi(t,0) \delta \overline{x}(0) \tag{29}$$

and equating (28) and (29), yields

$$\overline{E}_i(t) e^{\lambda_i t} = \Phi(t,0) \delta \overline{x}(0) \tag{30}$$

$$= \Phi(t,0) \overline{E}_i(0) \tag{31}$$

Since $\overline{E}_i(t)$ is a periodic vector, $\overline{E}_i(\tau) = \overline{E}_i(0)$, results in

$$\delta \overline{x}(\tau) = \overline{E}_i(\tau) e^{\lambda_i \tau} \tag{32}$$

$$= \overline{E}_i(0) e^{\lambda_i \tau} = \Phi(\tau,0) \overline{E}_i(0) \tag{33}$$

Rearranging (33) yields

$$[\Phi(\tau,0)-(e^{\lambda_i\tau})I]\overline{E}_i(0) = \overline{0} \tag{34}$$

which is an eigenvalue problem with the eigenvalues of $\Phi(\tau,0)$ being $e^{\lambda\tau}$. The eigenvectors of $\Phi(\tau,0)$ are, therefore, the vectors $\overline{E}_i(0)$.

To complete the solution, $\overline{E}_i(t)$ must be computed over one period. Since $\overline{E}_i(t)$ is periodic this determines $\overline{E}_i(t)$ for all time.

Combining equations (26) and (28) produces

$$\frac{d}{dt}(\overline{E}_i(t)e^{\lambda_i t}) = A(t)(\overline{E}_i(t)e^{\lambda_i t}) \tag{35}$$

Defining $\Lambda(t)$ as

$$\Lambda(t) = [\overline{E}_1(t)|\overline{E}_2(t)|...|\overline{E}_n(t)] \tag{36}$$

and incorporating $\Lambda(t)$ into (35) yields

$$\dot{\Lambda}(t)e^{Jt} + \Lambda(t)Je^{Jt} = A(t)\Lambda(t)e^{Jt} \tag{37}$$

and reducing further gives the differential equation

$$\dot{\Lambda}(t) + \Lambda(t)J = A(t)\Lambda(t) \tag{38}$$

Shelton used the Floquet theory described above to investigate the stability of the orbit. His work demonstrated, as did work previously done by Wiesel and Smith,

16

that the orbit was unstable. One of the system's Poincare exponents is a positive real root, while its conjugate is negative and the others are purely imaginary.

Table I. Poincare Exponents of Shelton's Orbit

Mode	Poincare Exponent
Planar	$0.0 - 1.0392i$
	$0.0 + 1.0392i$
Planar	$2.3921 + 0.0i$
	$-2.3921 + 0.0i$
Out of	$0.0 - 1.1247i$
Plane	$0.0 + 1.1247i$

In Shelton's orbit, the unstable exponent was designated λ_3 and is the root that his controller affected (Ref. 6). This gave the following equation when the controller was incorporated

$$\delta \dot{\overline{x}}(t) = A(t)\delta\overline{x}(t) + B(t)\overline{u}(t) \tag{39}$$

where $\overline{u}(t)$ is the control vector and $B(t)$ is used to designate to which states the control will be applied. In order to design the control system so only the unstable mode is affected by the controller, a new variable was defined.

$$\overline{\eta}(t) = \Lambda^{-1}(t)\,\overline{x}(t) \tag{40}$$

where $\overline{\eta}(t)$ is the modal vector and $\Lambda(t)$ is as defined in (36). Taking the time derivative of equation (40) yields

$$\delta \dot{\overline{x}}(t) = \dot{\Lambda}(t)\overline{\eta}(t) + \Lambda(t)\dot{\overline{\eta}}(t) \qquad (41)$$

Equating equations (39) and (41) and solving for $\dot{\overline{\eta}}(t)$, to get a differential equation in terms of the modal variables, results in

$$\dot{\overline{\eta}}(t) = \Lambda^{-1}(t)[A(t)\Lambda(t) - \dot{\Lambda}(t)]\overline{\eta}(t) + \Lambda^{-1}(t)B(t)\overline{u}(t) \qquad (42)$$

From (38)

$$\dot{\Lambda}(t) = A(t)\Lambda(t) - \Lambda(t)J \qquad (43)$$

substituting (43) in (42) produces

$$\dot{\overline{\eta}}(t) = J\overline{\eta}(t) + \Lambda^{-1}(t)B(t)u(t)$$

where J is the constant diagonal matrix of Poincare exponents. Since the system is diagonalized, feeding back a single unstable mode can be accomplished, making it relatively simple to implement the control.

Shelton implemented this control scheme by selecting $\overline{u}(t) = U = k\eta_3$ where k is the constant control gain. This *gave the following differential equation*

$$\begin{bmatrix} \dot{n}_1(t) \\ \dot{n}_2(t) \\ \dot{n}_3(t) \\ \dot{n}_4(t) \end{bmatrix} = \begin{bmatrix} \lambda_1 & & & n_1(t) \\ & \lambda_2 & 0 & n_2(t) \\ & 0 & \lambda_3 & n_3(t) \\ & & \lambda_4 & n_4(t) \end{bmatrix} + k\Lambda^{-1}(t)B\, n_3(t) \qquad (44)$$

The next chapter will discuss how, using Shelton's controlled system, a perturbation analysis was performed on the system and the controller was modified to ignore the oscillations forced on the orbit by the moon's eccentricity.

19

III. Perturbation Analysis

The solution of a differential equation of the sort

$$\dot{\overline{x}} = \overline{A} \, x + \overline{F} \tag{45}$$

contains a free (homogeneous) and a forced (particular) solution such that the total solution is of the form

$$\overline{x} = \overline{x}_h + \overline{x}_p \tag{46}$$

with $\overline{x}_p$ being the particular solution associated with a particular $\overline{F}$ (forcing function). Shelton's controller is designed to suppress both the free and forced components of the unstable mode. The controller was designed for the idealized case with the moon in a circular orbit. When Shelton's controller is implemented with the moon in its real, slightly eccentric, orbit the controller does not keep the satellite's orbit stable. In order to incorporate the perturbations caused by the moon, in the variational equations the lowest order perturbation terms evaluated on the periodic orbit will be included such that

$$\delta \dot{\overline{x}} = \{A(t) + \overline{B}K \Lambda^{-1}(t)\} \, \delta \overline{x} + \overline{P}(t) \tag{47}$$

where $K = \lceil 0 \; 0 \; k \; 0 \rceil$ when only η_3 is fed back, $\overline{P}(t)$ represents the perturbing effects of the moon's eccen-

20

tricity and $K\Lambda^{-1}(t)\ \overline{x}$ is the Shelton controller. Defining

$$A'(t) = A(t) + \overline{B}K\Lambda^{-1}(t) \tag{48}$$

and applying the Floquet transform to the modal variables for this controlled system (see section on Floquet theory), we find

$$\dot{\overline{\eta}}'(t) = J'\overline{\eta}'(t) + \Lambda'^{-1}(t)\ \overline{P}(t) \tag{49}$$

where the primes refer to the closed loop system, for a given gain k. This reduces the system (47) to constant coefficient form (Ref. 9).

The solution to the constant coefficient, linear differential equation is of the form (46). The forcing function $\overline{P}(t)$ was developed, to lowest order, using

$$\overline{P}(t) = \dot{\overline{x}}_R(t) - \dot{\overline{x}}_{P0}(t) \tag{50}$$

where $\dot{\overline{x}}_R(t)$ is the derivative of the solution to the variational equation with the moon's actual eccentricity included and $\dot{\overline{x}}_{P0}(t)$ is the derivative of the solution with the moon in its idealized, circular orbit.

$\Lambda'^{-1}\ \overline{P}(t)$ was obtained by evaluating (50) at thirty equally spaced time steps around the orbit while holding the

21

angle associated with the phase of the moon constant,
multiplying times $\Lambda'^{-1}(t)$, and then constructing a Fourier
series to represent the elements at each time step. Keeping
only the first N terms of the series yields

$$\Lambda'^{-1}(t)\ \overline{P}(t) = \sum_{i=0}^{N-1} \overline{A}'_i \cos(a_i) + \sum_{i=0}^{N-1} \overline{B}'_i \sin(a_i) \tag{51}$$

where

$$a_i = i w_0 t$$

This procedure was repeated until the Fourier series for a
number of small, constant step increments of the phase of the
moon from 0 to 2π radians were obtained. Another Fourier
series was then constructed from the coefficients, A'_i's and
B'_i's, of these Fourier series. The result was a double sum
Fourier series represented by

$$\Lambda'^{-1}(t)\overline{P}(t) = \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} [\overline{A}_{nm}\cos(a_n)\cos(b_m) + \overline{B}_{nm}\cos(a_n)\sin(b_m)$$

$$+ \overline{C}_{nm}\sin(a_n)\cos(b_m) + \overline{D}\sin(a_n)\sin(b_m)] \tag{52}$$

where the coefficients A_{nm}, B_{nm}, C_{nm}, and D_{nm} are the coef-
ficients resulting from the construction of the overall
Fourier series (52). The terms a_n and b_m are associated with
the phase of the orbit and the phase of the moon, respec-
tively, and are defined as

22

$$a_n = nw_0 t \tag{53}$$

$$b_m = m(w_E t + O_{10}) \tag{54}$$

where n and m are the summation indices, w_0 is the periodic orbit frequency, w_E is the frequency of the line of apsides of the lunar orbit, and O_{10} is a constant phase.

Since the product of two trigonometric functions can also be represented as the sum and difference of two trig functions the following relationships were used to expand $\Lambda'(t)\ \overline{P}(t)$.

$$\sin(a)\sin(b) = \frac{1}{2}\cos(a-b) - \frac{1}{2}\cos(a+b) \tag{55}$$

$$\cos(a)\cos(b) = \frac{1}{2}\cos(a-b) + \frac{1}{2}\cos(a+b) \tag{56}$$

$$\sin(a)\cos(b) = \frac{1}{2}\sin(a+b) + \frac{1}{2}\sin(a-b) \tag{57}$$

$$\cos(a)\sin(b) = \frac{1}{2}\sin(a+b) + \frac{1}{2}\sin(a-b) \tag{58}$$

Defining

$$\phi_{nm} = a + b = (nw_0 + mw_E)t + mQ_{10} \tag{59}$$

$$\psi_{nm} = a - b = (nw_0 - mw_E)t - mQ_{10} \tag{60}$$

and substituting (55) through (60) into (52) produces

23

$$\Lambda'(t)\overline{P}(t) = 1/2 \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} [(\overline{A}_{nm}+\overline{C}_{nm})\cos(\psi_{nm})+(\overline{A}_{nm}-\overline{C}_{nm})\cos(\phi_{nm})$$

$$+(\overline{B}_{nm}-\overline{D}_{nm})\sin(\psi_{nm})+(\overline{B}_{nm}+\overline{D}_{nm})\sin(\phi_{nm})] \qquad (61)$$

From (49) we see that

$$\dot{\overline{n}}'(t) = J'\overline{n}'(t) + \Lambda'^{-1}(t)\overline{P}(t) \qquad (62)$$

Since J' is of the form (Ref. 5)

$$J' = \begin{bmatrix} 0 & \alpha_1 & 0 & 0 \\ \alpha_2 & 0 & 0 & 0 \\ 0 & 0 & \alpha_3 & 0 \\ 0 & 0 & 0 & \alpha_4 \end{bmatrix} \qquad (63)$$

where the α's are the poincare exponents of the controlled system. Since $n_3(t)$ is used by the controller solving (62) for $n_{3p}(t)$ results in

$$\dot{n}_{3p}(t) = \alpha_3 n_{3p} + [\Lambda'^{-1}(t)\overline{P}(t)]_3 \qquad (64)$$

Assuming a solution of the same type as the forcing function and taking its derivative we obtain

$$n_{3p}(t) = \sum_{L=1}^{NM} [W_L\cos(\phi_L)+V_L\sin(\phi_L)+Y_L\cos(\psi_L)+Z_L\sin(\psi_L)] \qquad (65)$$

24

$$\dot{n}_{3p}(t) = \sum_{L=1}^{NM} \left[-W_L\dot{\phi}_L\sin(\phi_L) + V_L\dot{\phi}_L\cos(\phi_L) \right.$$

$$\left. -Y_L\dot{\psi}_L\sin(\psi_L) + Z_L\dot{\psi}_L\cos(\psi_L) \right] \tag{66}$$

where W_L, V_L, Y_L, and Z_L are the coefficients we are solving for and L is related to the nm designator in (61).

Substituting (61), (65) and (66) into (64) and equating like terms yields (L corresponding to a particular nm, $L = Mn + m + 1$)

$$\sin(\phi_L): \quad -W_L\dot{\phi}_L = \alpha_3 V_L + 1/2(B_{nm}+D_{nm}) \tag{67}$$

$$\cos(\phi_L): \quad V_L\dot{\phi}_L = \alpha_3 W_L + 1/2(A_{nm}-C_{nm}) \tag{68}$$

$$\sin(\psi_L): \quad -Y_L\dot{\psi}_L = \alpha_3 Z_L + 1/2(B_{nm}-D_{nm}) \tag{69}$$

$$\cos(\psi_L): \quad Z_L\dot{\psi}_L = \alpha_3 Y_L + 1/2(A_{nm}+C_{nm}) \tag{70}$$

Solving (67) through (70) for W_L, V_L, Y_L and Z_L and substituting in

$$\dot{\phi}_L = nw_0 + mw_E \tag{71}$$

$$\dot{\psi}_L = nw_0 - mw_E \tag{72}$$

we obtain the coefficients for the particular solution of $n_3(t)$ as follows

$$W_L = \frac{-\alpha_3 V_L - 1/2 (B_{nm} + D_{nm})}{(nw_0 + mw_E)} \qquad (73)$$

$$V_L = \frac{\alpha_3 W_L + 1/2 (A_{nm} - C_{nm})}{(nw_0 + mw_E)} \qquad (74)$$

$$Y_L = \frac{-\alpha_3 Z_L - 1/2 (B_{nm} - D_{nm})}{(nw_0 - mw_E)} \qquad (75)$$

$$Z_L = \frac{\alpha_3 Y_L + 1/2 (A_{nm} + C_{nm})}{(nw_0 - mw_E)} \qquad (76)$$

Numerically evaluating (73) through (76) and using them with (65) gives us a solution to $n_{3p}(t)$. To insure that this solution was correct $n_{3p}(t_1)$ was numerically calculated and using the approximation

$$\dot{n}_{3p}(t_1) = \frac{n_{3p}(t_1) - n_{3p}(t_1 + \Delta t)}{\Delta t} \qquad (77)$$

and substituting these values into

$$P_3(t_1) = \Lambda(t_1) [n_{3p}(t_1) - \alpha_3 \dot{n}_{3p}(t_1)] \qquad (78)$$

This calculated value was compared with the value for $P_3(t)$ calculated from (50). The numbers matched to six decimal places using a $\Delta t = 10^{-6}$ sec. This makes me resonably sure that the correct solution for $n_{3p}(t)$ was formulated. The

26

derivations of $n_{1p}(t)$, $n_{2p}(t)$ and $n_{\Delta p}(t)$ are contained in Appendix A.

IV. Implementation of the Modified Controller

Having solved for the particular solution due to the
perturbation effects of the moon Shelton's controller was
modified to incorporate this particular solution.   Shelton's
controller stabilizes the idealized orbit by zeroing out the
difference between the satellite's true orbit and the stable
controlled periodic orbit.   When the moon's perturbation ef-
fects are added in, the controlled orbit is no longer
stabilized. The Fourier analysis of the perturbing force
showed it to be purely oscillatory, with no secular terms, to
first order.   If we could calculate these terms and subtract
them from the controlled term we could stabilize the orbit
with the moon's proper dynamics involved.

The Shelton controller stabilizes the idealized
system by calculating the control term given in (44)

$$CT. = k\Lambda^{-1}(t)B\eta_3(t) \qquad (79)$$

which, with the moon in its actual orbit, yield

$$CT. = k\Lambda^{-1}(t)B[\eta_{3H}(t) + \eta_{3p}(t)] \qquad (80)$$

driving the system unstable.   Having solved for the par-
ticular solution (65) the controller was implemented by sub-
tracting off $\eta_{3p}(t)$ which yields

28

$$CT. = k\Lambda^{-1}(t)Bn_{3H}(t) \tag{P1}$$

This system produces a new stabilized orbit which is not the idealized periodic orbit that Shelton derived.

V. Results and Conclusions

Evaluation of the Controller

As stated in chapter IV of this report, the modified controller design was based on a linearized set of equations including a first order approximation of the forcing function caused by the eccentricity of the moon's orbit. To evaluate the controller both the actual integrated solution with the modified controller operating and the particular solution,

$\overline{\eta}_p(t)$, were plotted to see if the integrated solution would track the calculated particular solution (Figure 3). All four modal variables were plotted to insure that the calculated solution was indeed tracking the integrated solution for each variable. As can be seen in Figure 3 the calculated and integrated solutions match very well.

During the sixth orbit the integrated solution becomes unstable (Figure 4). This occurs due to a combination of two factors, the gain selected for implementation of Shelton's controller, k=.459, was small and the particular solution was larger than expected. Because the particular solution was so much larger than expected, the error in the calculated term is also quite appreciable. This is because the perturbation effects were assumed to be first order and were evaluated with respect to the periodic orbit, which would have been a good assumption if the particular solution
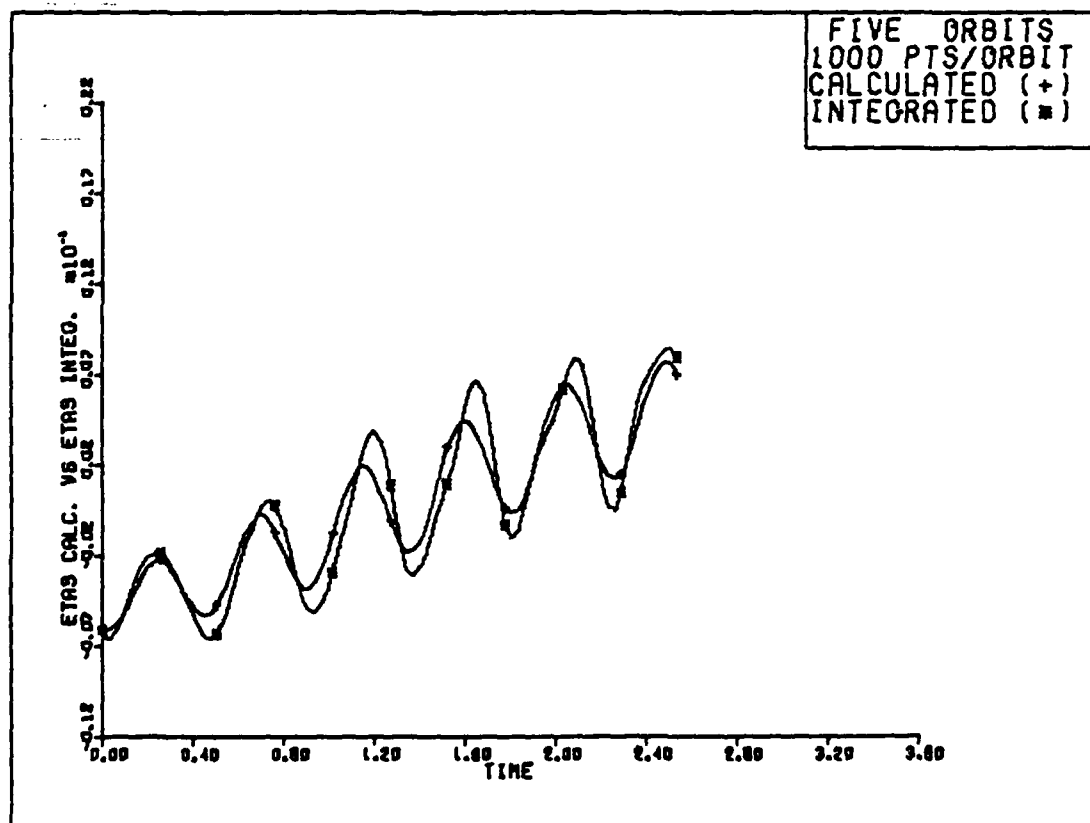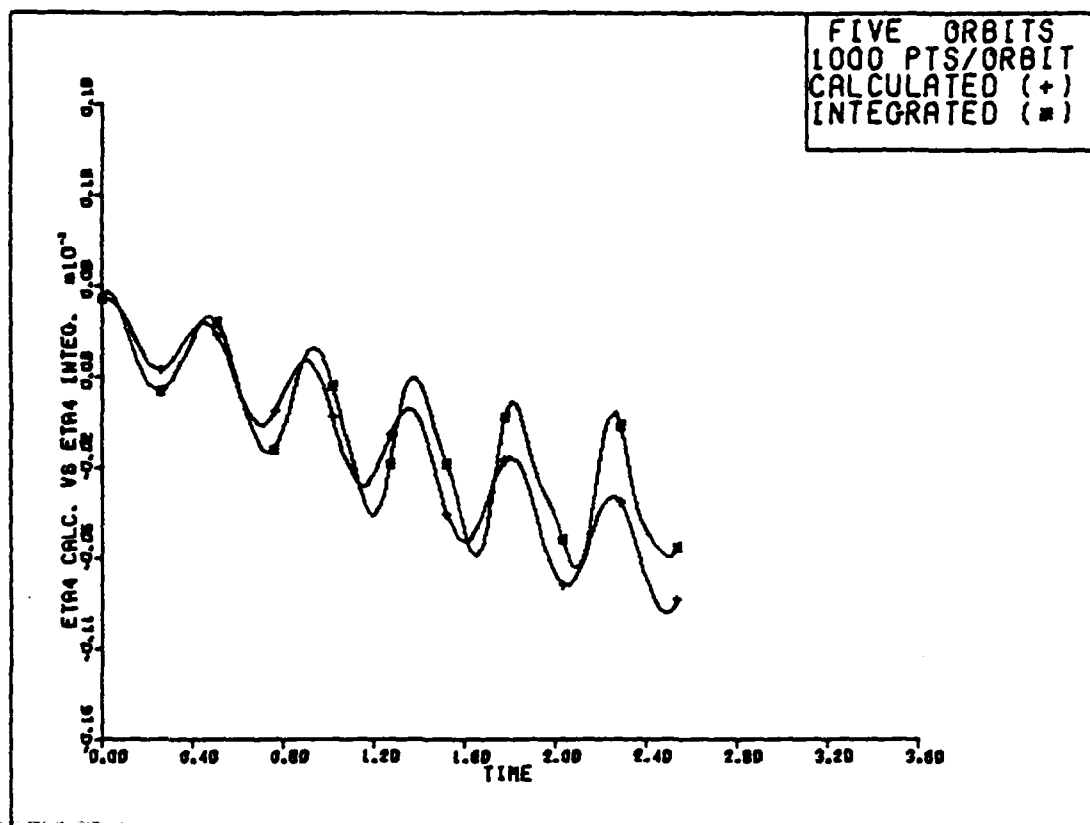
30
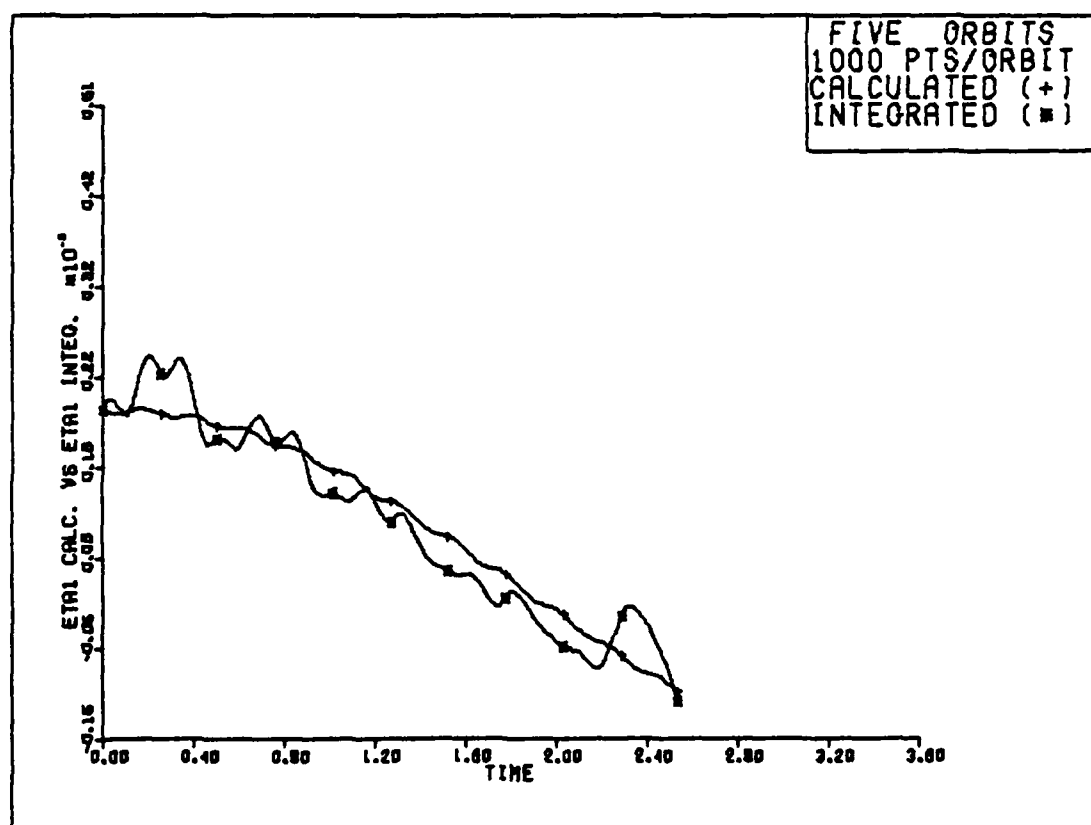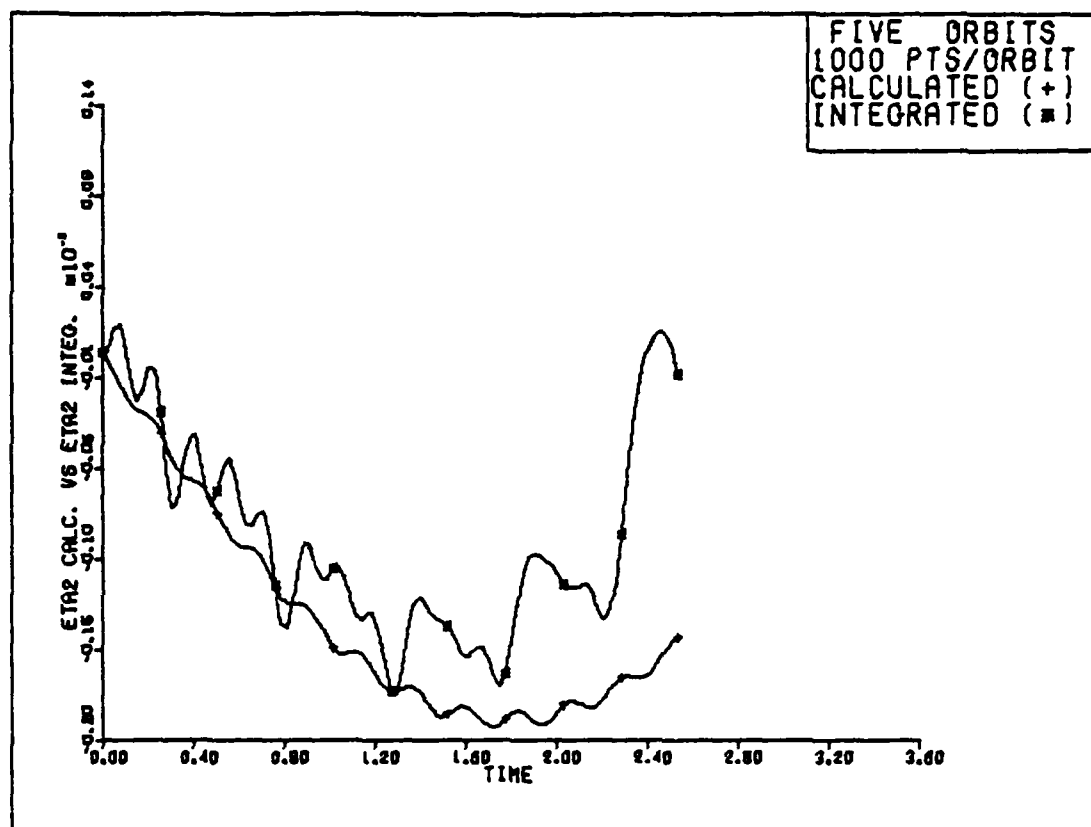
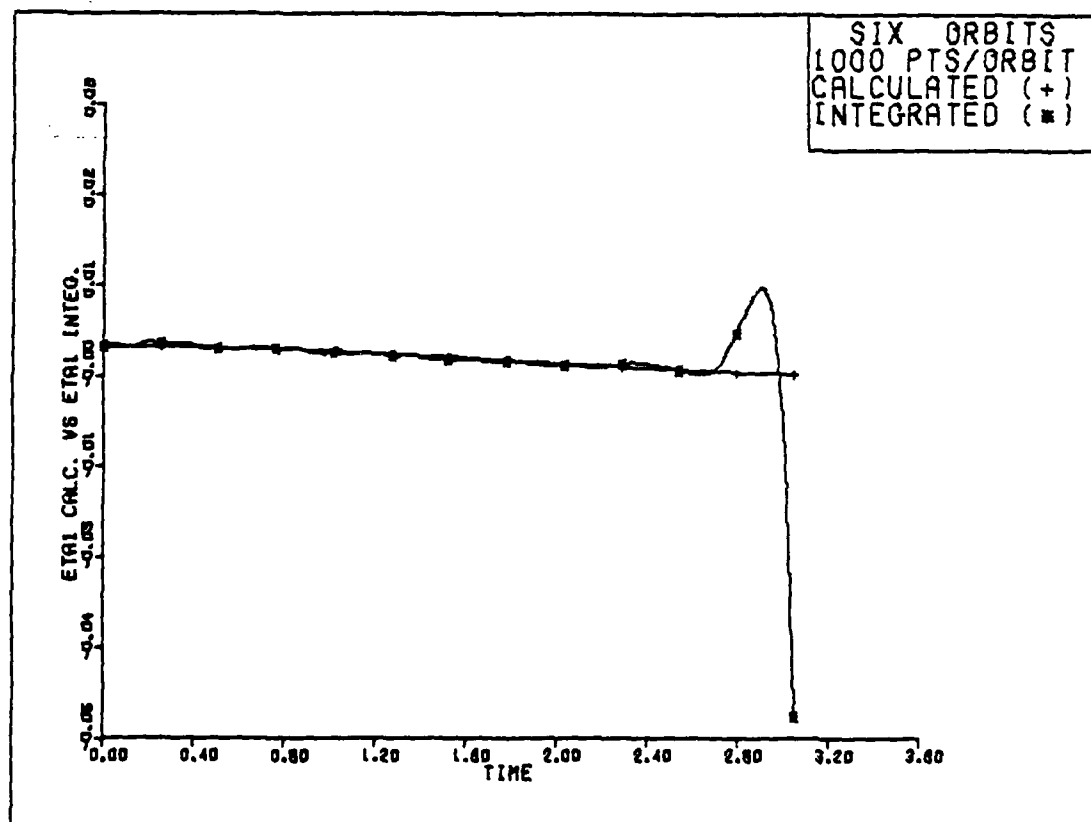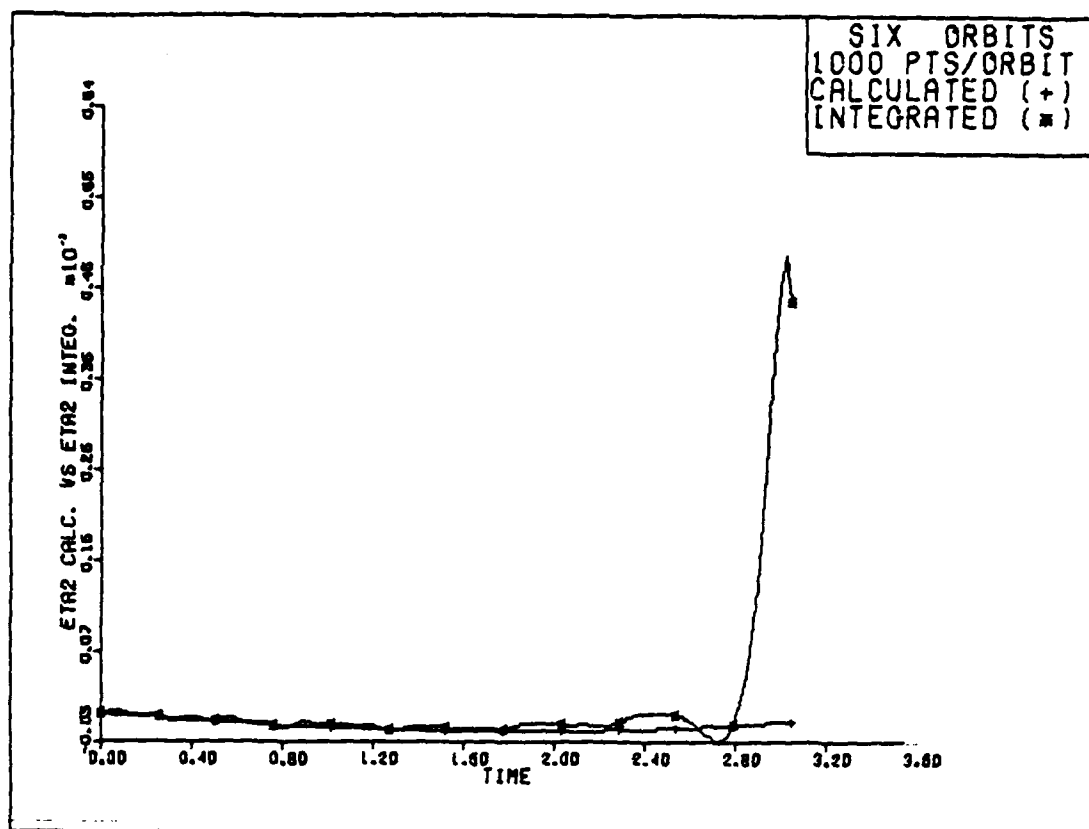Figure 3(a) ETA vs Time

31

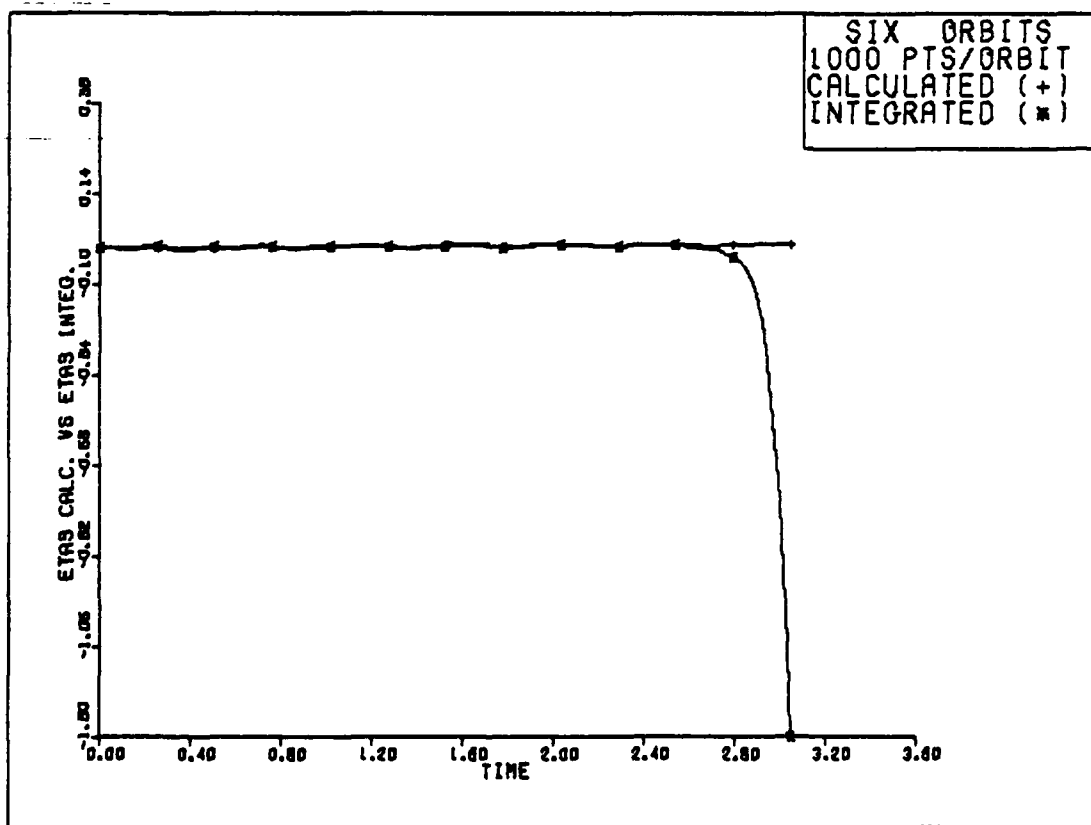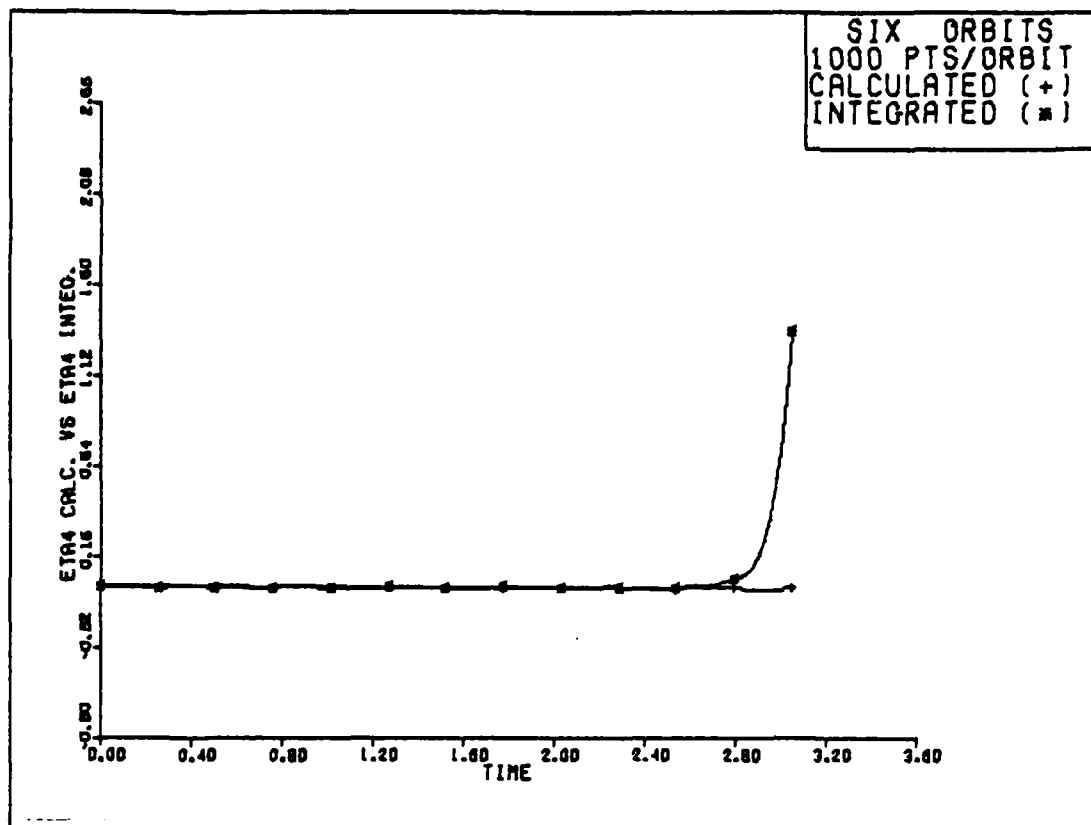Figure 3(b) ETA vs Time

32

Figure 4(a)   ETA vs Time

33

Figure 4(b)   ETA vs Time

34

were two orders of magnitude smaller. A phase portrait for the Shelton Controller with a gain of 0.5 is depicted in Figure 5 (Ref 6). As can be seen in this figure some of the trajectories, on the order of $n_3 = n_4 = 10^{-3}$, do not converge to the origin. The combination of factors mentioned above put this solution on one of the trajectories that is unstable.

Due to the combination of factors described previously the system is not stable beyond six orbits. This situation could possibly have been avoided if the particular solution, $n_{p3}(t)$, used in the controller was more accurate or if a larger gain was selected for the original controller at the start of the analysis. Selecting a larger gain would have increased the stable operating region of the controller. However, there was no way of actually knowing ahead of time how large or accurate the particular solution would be.

In spite of the failure of the controller to operate beyond six orbits, it did exhibit excellent control characteristics up to the time it became unstable. In Figure 6 we can see that the integrated and calculated solutions track very well. In this case the integrated solution was started on the calculated solution. To see if the controller would drive the integrated solution to the calculated solution, I started the integrated solution off of the calculated solution and the controller did drive it back to the calculated solution (Figure 7). Figure 8 shows the difference between the calculated and integrated solution for five orbits and
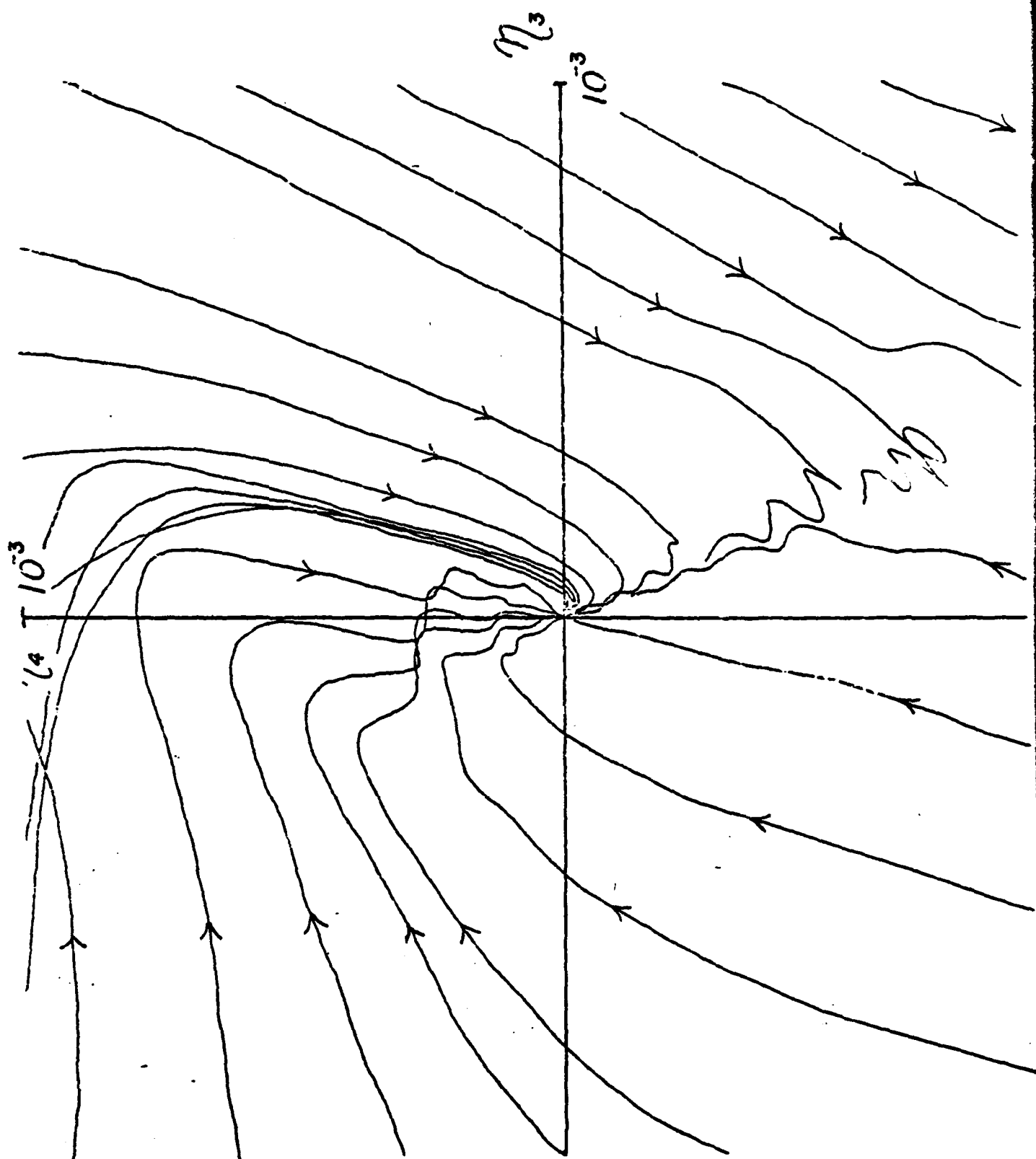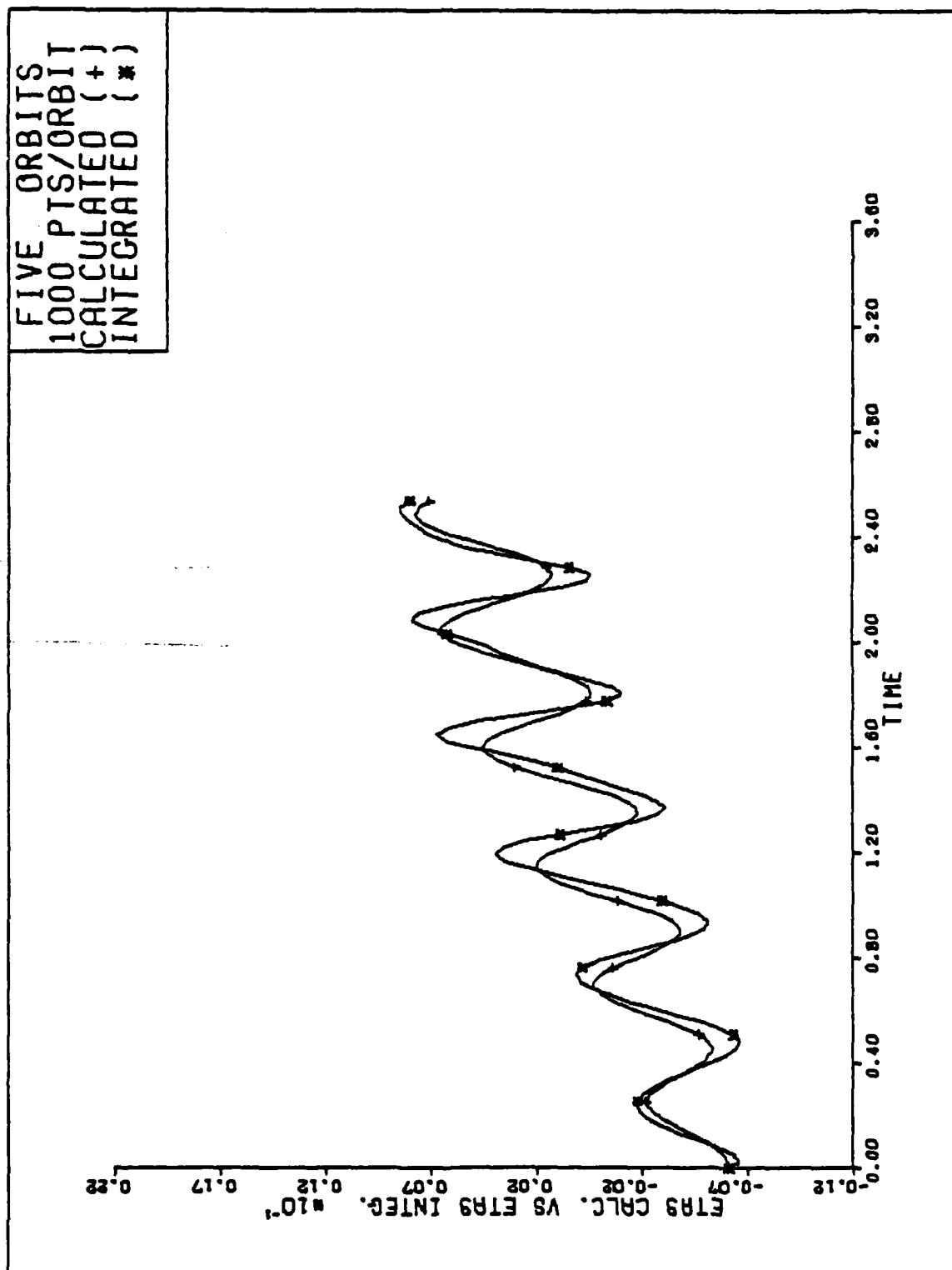
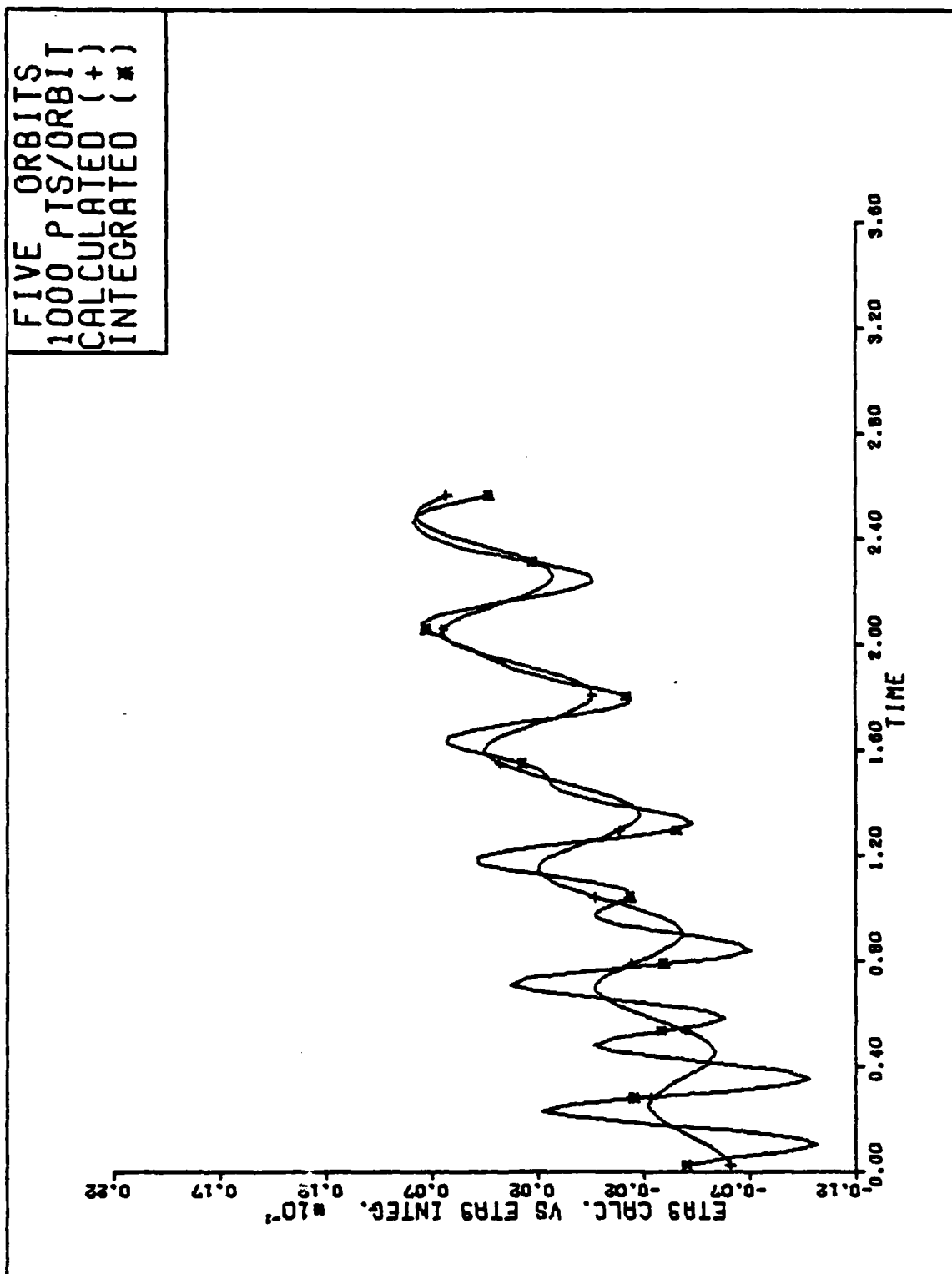Figure 5   ETA3-4 Phase Portrait

36

Figure 6 ETA3 Started on Orbit

37

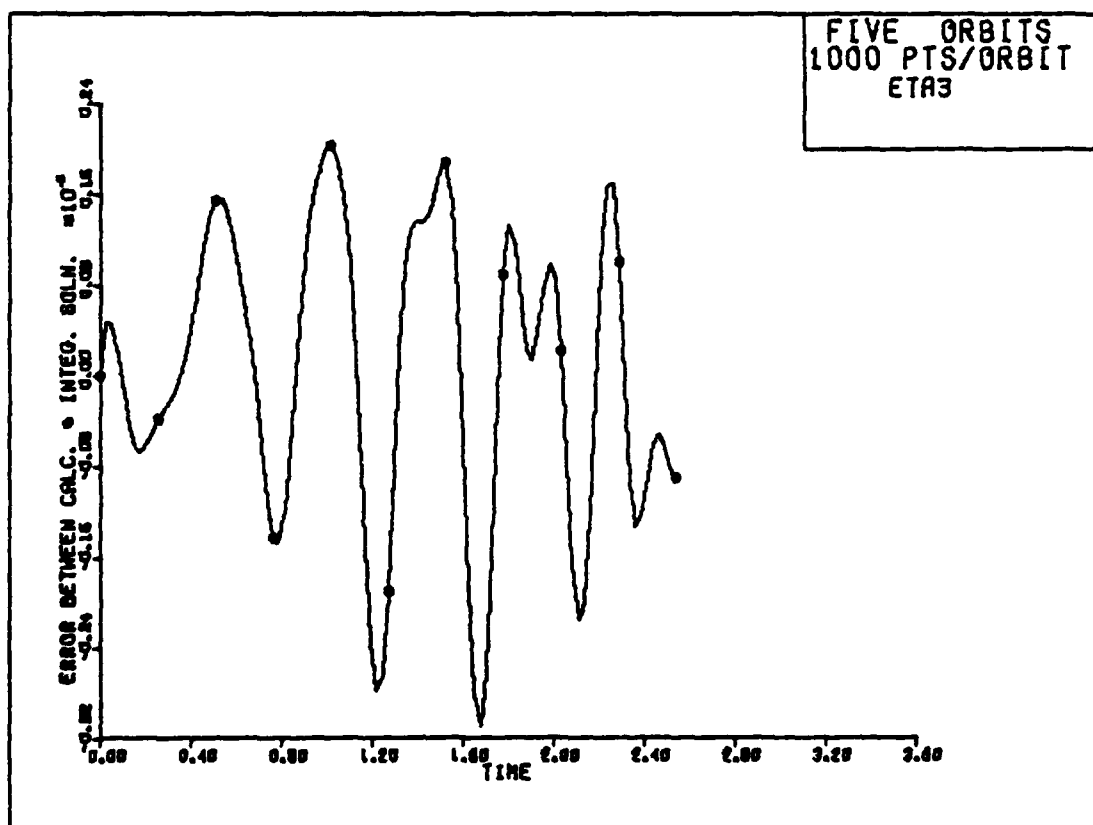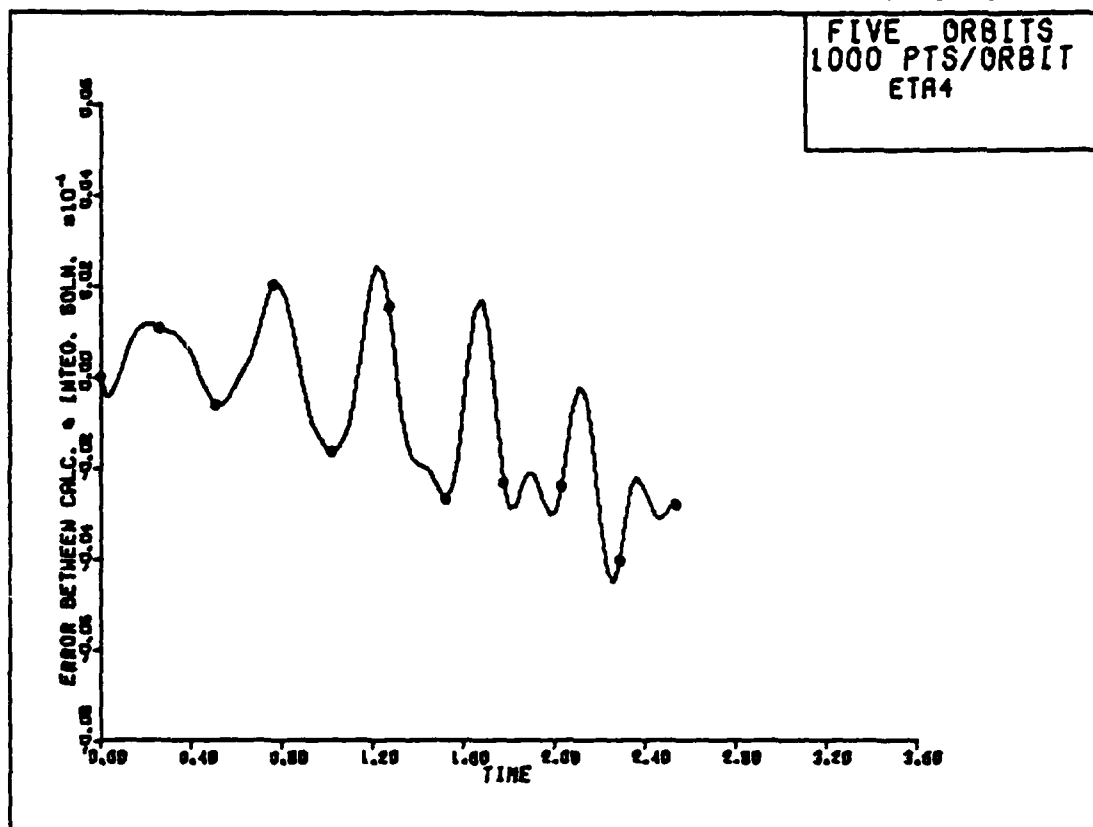Figure 7   ETA3 Started off Orbit

38

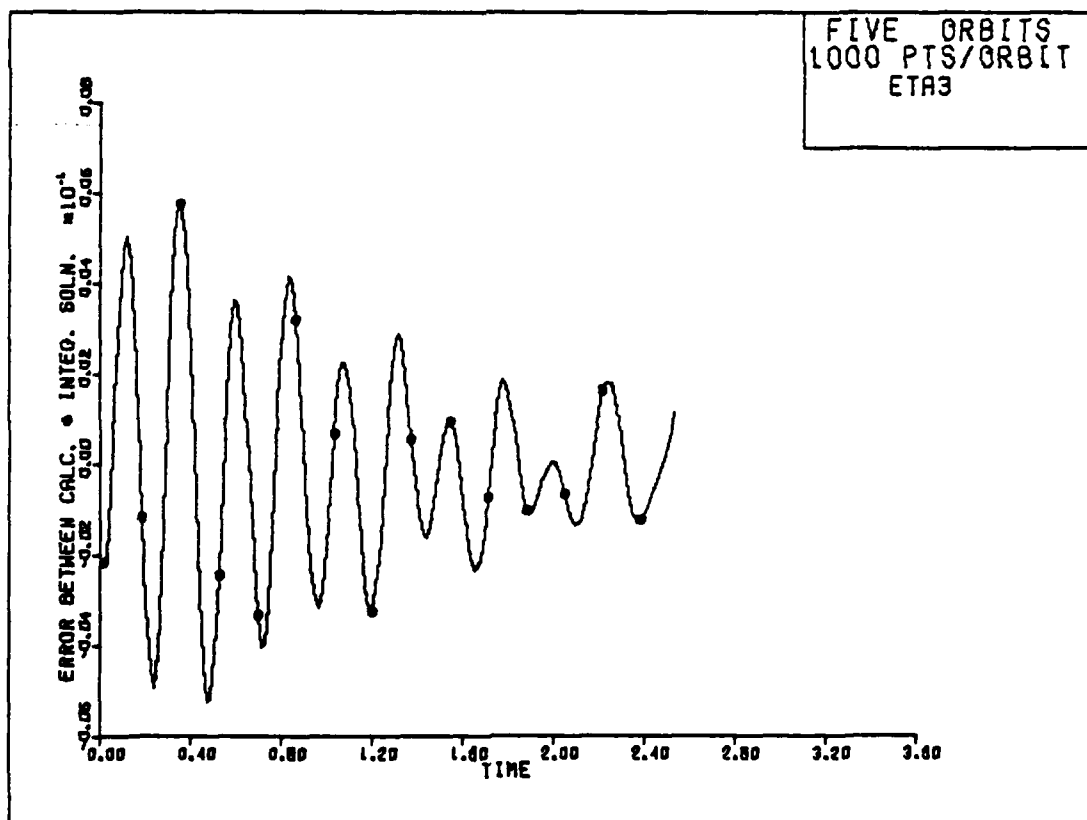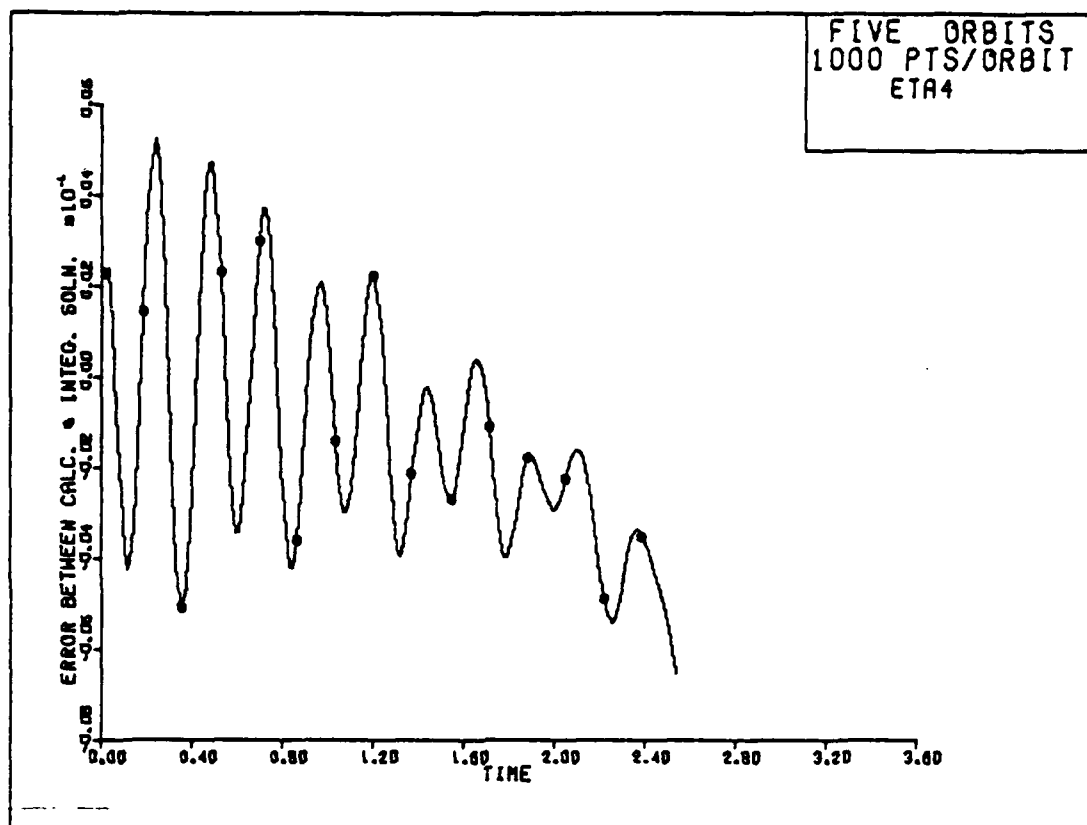Figure 8(a)   Difference Between Calculated and Integrated Solutions

39

Figure 8(b)   Difference Between Calculated and Integrated Solutions

40

Figure 9 shows the amount of control being expended to maintain the orbit. The integrated control gains (ICG) shown in Figure 9 are defined as

$$ICG(t) = \frac{1}{t} \int_0^t |control\ acc.|\ d\gamma$$

which is equal to the average control acceleration required for system operation. Expressing these values in terms of g's, yields an average control cost of approximately $10^{-7}g$ which is comparable to a typical earth-synchronous satellite's control cost of $10^{-7}g$ (Ref. 2:63).

Conclusions

As shown in Figures 7, 8 and 9 the modfied controller does work very well over this five orbit period. Because the initial selection of gain for the Shelton controller was too low I did not achieve the long term, low cost solution I was expecting to develop. However, because of the initial well behaved aspect of the controller, I believe the method is sound. In order to fully develop this control system and prove that it is feasible, I recommend that a follow up study be done using a higher gain and also include the effects of the inclination of the moon's orbit.
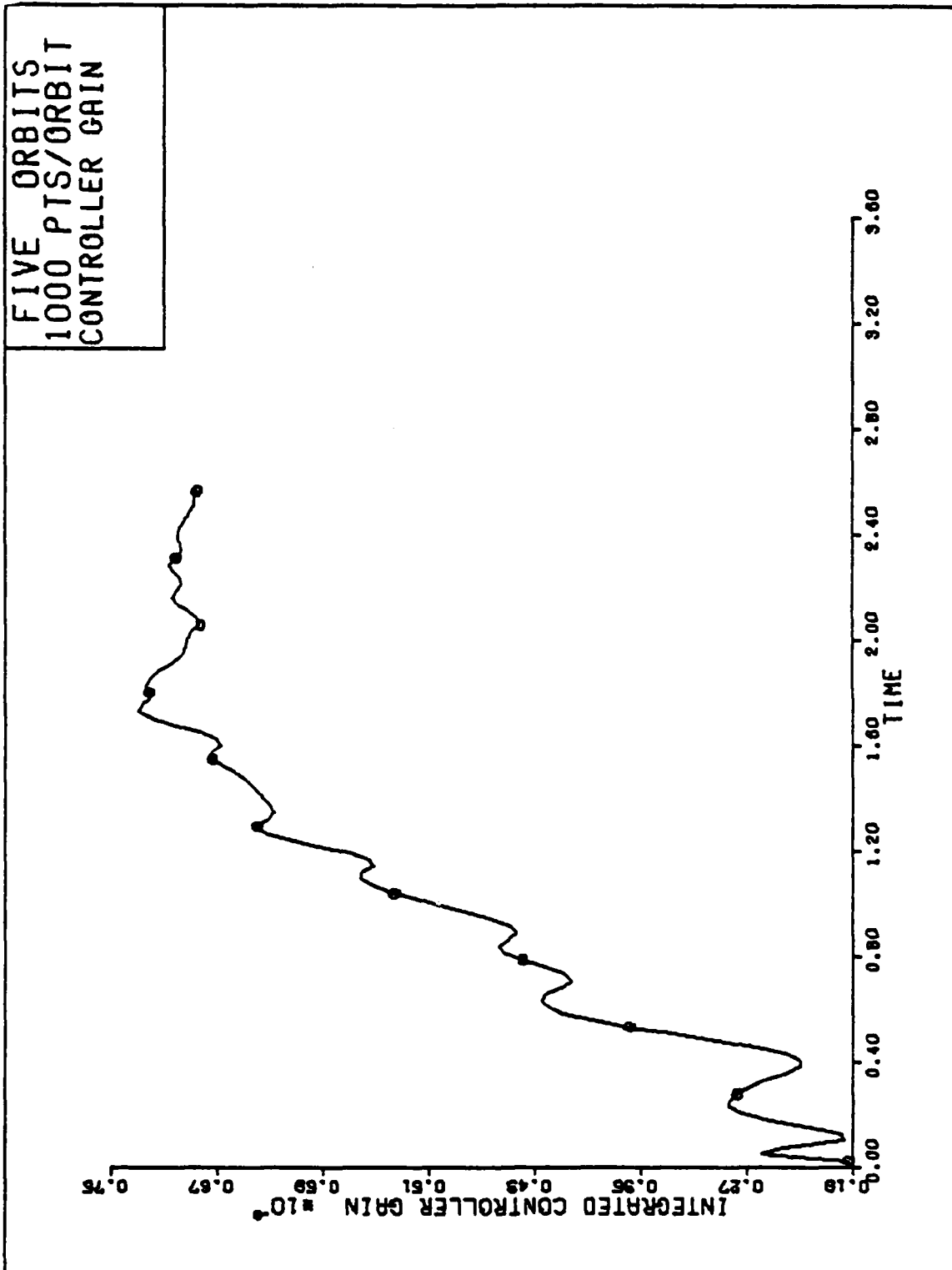
Figure 9(a) Controlled System (Started on Orbit)

42

FIVE ORBITS
1000 PTS/ORBIT
CONTROLLER GAIN

INTEGRATED CONTROLLER GAIN *10²

0.22 0.20 0.18 0.16 0.14 0.12 0.10 0.08

0.00 0.40 0.80 1.20 1.60 2.00 2.40 2.80 3.20 3.60

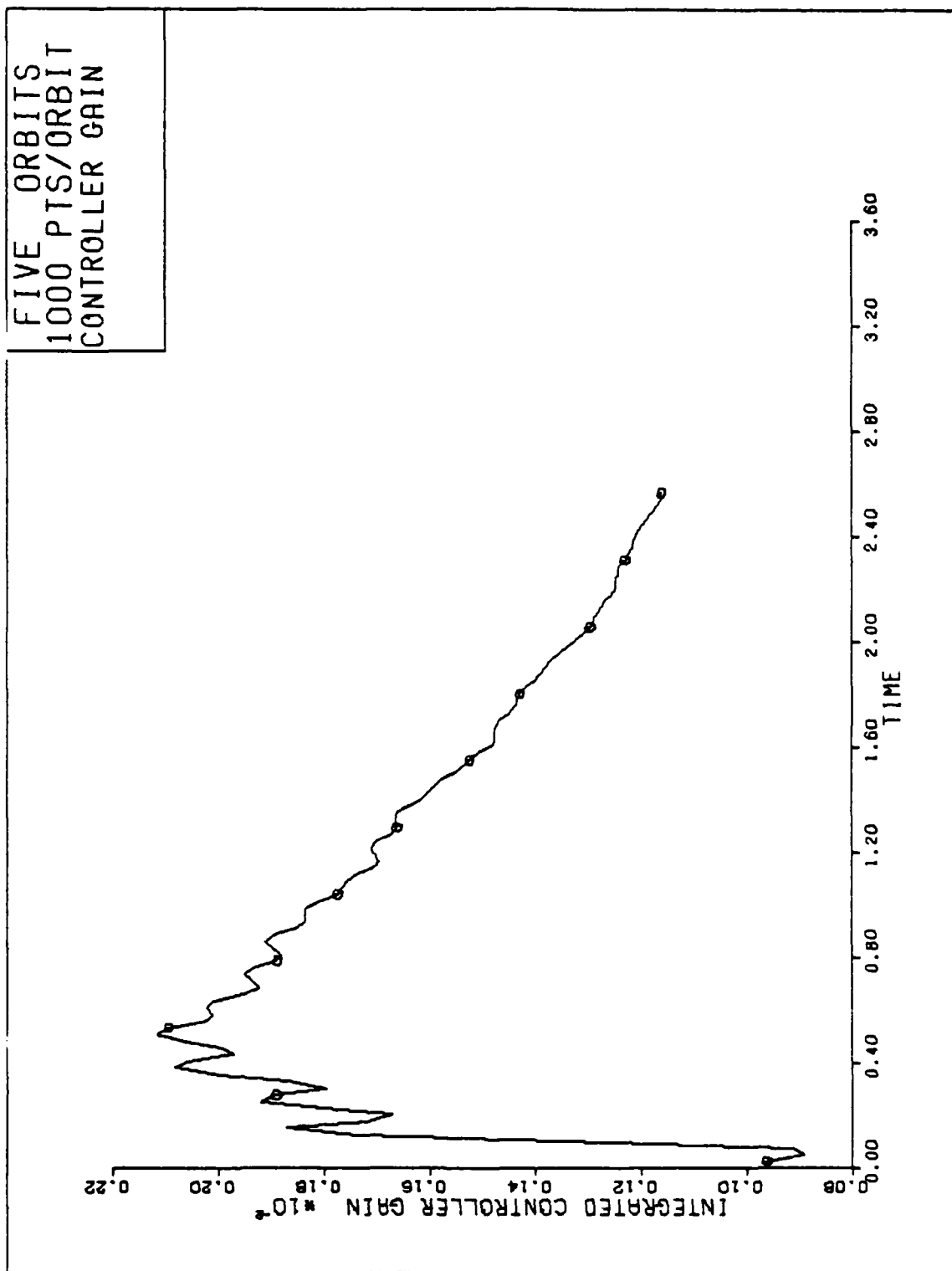TIME

Figure 9(b) Controlled System (Started off Orbit)

43

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Appendix A

Derivation of the Particular Solution

In deriving the particular solution to (62) we can see that the form of the solutions $n_{3p}(t)$ and $n_{4p}(t)$ are the same except for a change of indices. If I change the 3's to 4's in (64) on page 24, and follow through the rest of the derivation on pages 24 through 26, the solutions are generically the same.

The differential equations involving $n_{1p}(t)$ and $n_{2p}(t)$ are coupled and of the form

$$\dot{n}_{1p}(t) = \alpha_1 n_{2p}(t) + [\Lambda'(t)\overline{P}(t)]^{-1}_1 \tag{82}$$

$$\dot{n}_{2p}(t) = \alpha_2 n_{1p}(t) + [\Lambda'(t)\overline{P}(t)]^{-1}_2 \tag{83}$$

Taking the time derivative of (82) and substituting (83) produces

$$\ddot{n}_{1p}(t) = \alpha_1 \alpha_2 n_{1p}(t) + \alpha_1 F_2 + \dot{F}_1 \tag{84}$$

where $\dot{F}_1$ is the time derivative of $[\Lambda'(t)\overline{P}(t)]^{-1}_1$ and

$$F_2 = [\Lambda'(t)\overline{P}(t)]^{-1}_2 \tag{85}$$

Assuming a solution to (84) of the form

45

$$\eta_{1p}(t) = \sum_{L=1}^{NM} \lceil W_L \cos(\phi_L) + V_L \sin(\phi_L) + Y_L \cos(\psi_L) + Z_L \sin(\psi_L) \rceil \qquad (86)$$

taking time derivatives of (86) yields

$$\dot{\eta}_{1p}(t) = \sum_{L=1}^{NM} \lceil -W_L \dot{\phi}_L \sin(\phi_L) + V_L \dot{\phi}_L \cos(\phi_L) - Y_L \dot{\psi}_L \sin(\psi_L) + Z_L \dot{\psi}_L \cos(\psi_L) \rceil \quad (87)$$

$$\ddot{\eta}_{1p}(t) = \sum_{L=1}^{NM} \lceil -W_L \dot{\phi}_L^2 \cos(\phi_L) - V_L \dot{\phi}_L^2 \sin(\phi_L) - Y_L \dot{\psi}_L^2 \cos(\psi_L) - Z_L \dot{\psi}_L^2 \sin(\psi_L) \rceil \quad (88)$$

with

$$\phi = (nw_0 + mw_E)t + m\Omega_{10} \qquad (89)$$

$$\psi = (nw_0 + mw_E)t - m\Omega_{10} \qquad (90)$$

$$\dot{\phi} = nw_0 + mw_E \qquad (91)$$

$$\dot{\psi} = nw_0 - mw_E \qquad (92)$$

$$\ddot{\phi} = \ddot{\psi} = 0 \qquad (93)$$

taking the time derivative of $\lceil \Lambda'(t)\overline{P}(t) \rceil_1^{-1}$, from (61) page 24, yields

$$\dot{F}_1 = \frac{1}{2} \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} \lceil -\dot{\psi}_{nm}(A_{nml}+C_{nml})\sin(\psi_{nm}) - \dot{\phi}_{nm}(A_{nml}-C_{nml})\sin(\phi_{nm})$$

$$+ \dot{\psi}_{nm}(B_{nml}-D_{nml})\cos(\psi_{nm}) + \dot{\phi}_{nm}(B_{nml}+D_{nml})\cos(\phi_{nm}) \rceil \qquad (94)$$

46

with $[\Lambda'(t)\bar{P}(t)]_2^{-1}$ equal to

$$F_2 = \frac{1}{2} \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} [(A_{nm2}+C_{nm2})\cos(\psi_{nm}) + (A_{nm2}-C_{nm2})\cos(\phi_{nm})$$

$$+ (B_{nm2}-D_{nm2})\sin(\psi_{nm}) + (B_{nm2}+D_{nm2})\sin(\phi_{nm})] \qquad (95)$$

taking (87) through (95), substituting them into (84) and equating like terms produces

$\sin(\phi_L)$:

$$-V_L(nw_0+mw_E)^2 = \alpha_1\alpha_2 V_L + \frac{1}{2}\alpha_1(B_{nm2}+D_{nm2}) - (nw_0+mw_E)(A_{nm1}-C_{nm1}) \qquad (96)$$

$\cos(\phi_L)$:

$$-W_L(nw_0+mw_E)^2 = \alpha_1\alpha_2 W_L + \frac{1}{2}\alpha_1(A_{nm2}-C_{nm2}) + (nw_0+mw_E)(B_{nm1}+D_{nm1}) \qquad (97)$$

$\sin(\psi_L)$:

$$-Z_L(nw_0+mw_E)^2 = \alpha_1\alpha_2 Z_L + \frac{1}{2}\alpha_1(B_{nm2}-D_{nm2}) - (nw_0-mw_E)(A_{nm1}+C_{nm1}) \qquad (98)$$

$\cos(\psi_L)$:

$$-Y_L(nw_0-mw_E)^2 = \alpha_1\alpha_2 Y_L + \frac{1}{2}\alpha_1(A_{nm2}+C_{nm2}) + (nw_0-mw_E)(B_{nm1}-D_{nm1}) \qquad (99)$$

solving for the coefficients of the particular solution from (96) through (99) we obtain

$$W_{1L} = - \frac{1/2\,\alpha_1(A_{nm2}-C_{nm2}) + 1/2(nw_0+mw_E)(B_{nm1}+D_{nm1})}{[\alpha_1\alpha_2 + (nw_0+mw_E)^2]} \qquad (100)$$

47

$$V_{1L} = - \frac{1/2\ \alpha_1 (B_{nm2}+D_{nm2}) - 1/2(nw_0+mw_E)(A_{nm1}-C_{nm1})}{\lceil \alpha_1 \alpha_2 + (nw_0+mw_E)^2 \rceil} \quad (101)$$

$$Y_{1L} = - \frac{1/2\ \alpha_1 (A_{nm2}+C_{nm2}) + 1/2(nw_0-mw_E)(B_{nm1}-D_{nm1})}{\lceil \alpha_1 \alpha_2 + (nw_0-mw_E)^2 \rceil} \quad (102)$$

$$Z_{1L} = - \frac{1/2\ \alpha_1 (B_{nm2}-D_{nm2}) - 1/2(nw_0+mw_E)(A_{nm1}+C_{nm1})}{\lceil \alpha_1 \alpha_2 + (nw_0-mw_E)^2 \rceil} \quad (103)$$

Numerically evaluating (100) through (103) and using them
with (86) gives us a solution to $n_{1p}(t)$. Equations (100)
through (103) can be used to provide the coefficients for
$n_{2p}(t)$ by switching the sub-indexes, 1 and 2.

VITA

Dennis W. Ehrler was born May 15, 1955 in Shreveport, Louisiana. In December, 1961, he and his family moved to a farm near Turtle Lake, Wisconsin where he attended the Turtle Lake school system. In May, 1973, he was graduated from Turtle Lake High school and entered into the U. S. Air Force Academy in June, 1973. He was graduated from the USAFA with a Bachelor of Science degree in Astronautical Engineering in 1977.

In July, 1977, Ehrler was assigned to the 6595th Missle Test Group at Vandenberg AFB, California, and spent three years as a test controller for experimental reentry vehicles. In September, 1979, he was graduated from the University of Southern California with a Master of Science degree in Systems Engineering.

In June, 1980, Ehrler was reassigned to the Air Force Institute of Technology School of Engineering at Wright-Patterson AFB, Ohio, and began studies toward a Master of Science degree in Astronautical Engineering. After graduation, Ehrler will be assigned to NASA'S Lyndon B. Johnson Space Center in Houston and will work on operational test and evaluation of the Space Shuttle.

Ehrler is married to the former Lisa M. Schickler of Rochester, New York.

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A Fourier representation of the perturbation effects of the moon's
eccentricity on a satellite in orbit about L3 was costructed and verified
numerically. It was then incorporated in a modal control scheme for
linear systems that are time periodic which was developed by Shelton.
The feasibility of this modified control scheme was then verified by
computer simulation by controlling the actual non-linear motions of the
satellite with the modified controller.

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